

**Year 2007**  
**VCE**  
**Specialist Mathematics**  
**Solutions**  
**Trial Examination 1**



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## Question 1

$\frac{x}{e^{2y}} - 5y + 6x^2 + 3 = 0$  taking  $\frac{d}{dx}$  of each term ( implicit differentiation )

$$\frac{d}{dx}(xe^{-2y}) - \frac{d}{dx}(5y) + \frac{d}{dx}(6x^2) + \frac{d}{dx}(3) = 0 \quad \text{M1}$$

product rule in the first term

$$x \frac{d}{dy}(e^{-2y}) \frac{dy}{dx} + e^{-2y} \frac{d}{dx}(x) - \frac{d}{dy}(5y) \frac{dy}{dx} + \frac{d}{dx}(6x^2) + \frac{d}{dx}(3) = 0$$

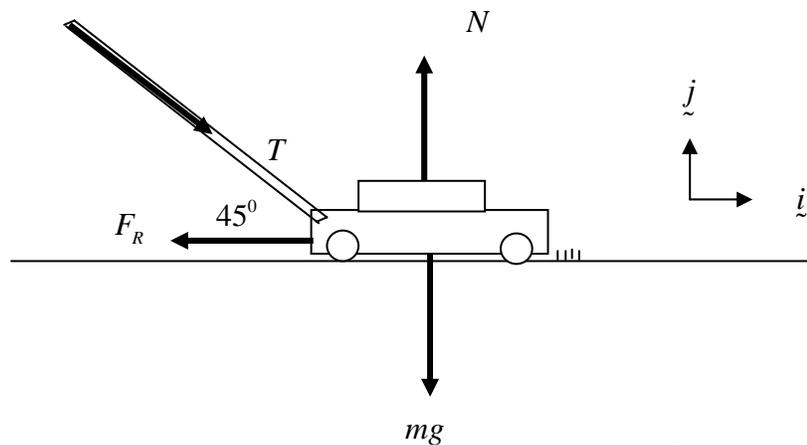
$$-2xe^{-2y} \frac{dy}{dx} + e^{-2y} - 5 \frac{dy}{dx} + 12x = 0 \quad \text{M1}$$

$$\frac{dy}{dx}(2xe^{-2y} + 5) = e^{-2y} + 12x$$

$$\frac{dy}{dx} = \frac{e^{-2y} + 12x}{2xe^{-2y} + 5} \quad \text{A1}$$

## Question 2

a.



for all the forces A1

b. now  $T = 12\sqrt{2}$   $m = 10\text{kg}$   $g = 9.8$   $\mu = \frac{1}{7}$   $\theta = 45^\circ$

resolving horizontally to the lawn in the  $\hat{i}$  direction.

$$(1) T \cos(\theta) - F_R = 0$$

$$\text{from (1) } F_R = T \cos(\theta) = 12\sqrt{2} \cos(45^\circ) = 12 \quad \text{A1}$$

resolving perpendicular to the lawn in the  $\hat{j}$  direction.

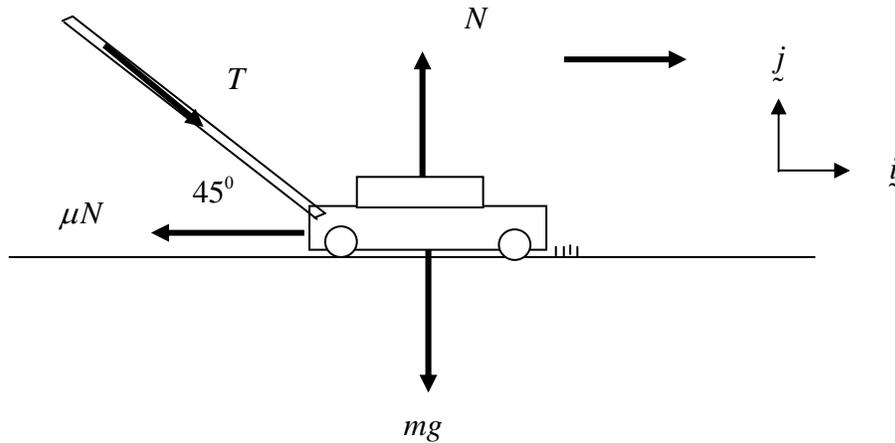
$$(2) N - T \sin(\theta) - mg = 0$$

$$\text{from (2) } N = mg + T \sin(\theta) = 10 \times 9.8 + 12\sqrt{2} \sin(45^\circ) = 98 + 12 = 110$$

$$\text{and } \mu N = \frac{110}{7} = 15 \frac{5}{7}$$

since  $\mu N > F_R$  it is **not** on the point of moving A1

c.



now  $T = \frac{49\sqrt{2}}{2}$   $m = 10\text{kg}$   $g = 9.8$   $\mu = \frac{1}{7}$   $\theta = 45^\circ$   $a = ?$

resolving horizontally to the lawn in the  $\hat{i}$  direction.

$$(1) \quad T \cos(\theta) - \mu N = ma \quad \text{M1}$$

resolving perpendicular to the lawn in the  $\hat{j}$  direction.

$$(2) \quad N - T \sin(\theta) - mg = 0 \quad \text{from (2) } N = T \sin(\theta) + mg \text{ into (1)}$$

$$ma = T \cos(\theta) - \mu(T \sin(\theta) + mg)$$

A1

$$ma = T(\cos(\theta) - \mu \sin(\theta)) - \mu mg$$

$$a = \frac{T}{m}(\cos(\theta) - \mu \sin(\theta)) - \mu g$$

$$a = \frac{49\sqrt{2}}{20} \left( \cos(45^\circ) - \frac{1}{7} \sin(45^\circ) \right) - \frac{9.8}{7}$$

$$a = \frac{49\sqrt{2}}{20} \left( \frac{1}{\sqrt{2}} - \frac{1}{7\sqrt{2}} \right) - 1.4$$

$$a = \frac{49\sqrt{2}}{20} \times \frac{6}{7\sqrt{2}} - 1.4$$

$$a = 2.1 - 1.4$$

$$a = 0.7 \text{ m/s}^2$$

A1

**Question 3**

$$\frac{dy}{dx} = \frac{3x-5}{\sqrt{9-4x^2}} \quad \text{integrating with respect to } x$$

$$y = \int \frac{3x-5}{\sqrt{9-4x^2}} dx \quad \text{separating into two integrals}$$

$$y = \int \frac{3x}{\sqrt{9-4x^2}} dx - \int \frac{5}{\sqrt{9-4x^2}} dx \quad \text{M1}$$

in the first integral let  $u = 9 - 4x^2$ ,  $\frac{du}{dx} = -8x$

in the second integral let  $v = 2x$ ,  $\frac{dv}{dx} = 2$

$$y = -\frac{3}{8} \int u^{\frac{1}{2}} du - \frac{5}{2} \int \frac{1}{\sqrt{9-v^2}} dv \quad \text{M1}$$

$$y = -\frac{3}{4} u^{\frac{1}{2}} - \frac{5}{2} \sin^{-1}\left(\frac{v}{3}\right) + C$$

$$y = -\frac{3}{4} \sqrt{9-4x^2} - \frac{5}{2} \sin^{-1}\left(\frac{2x}{3}\right) + C \quad \text{A2}$$

to find  $C$  use  $y(0) = 0$

$$0 = -\frac{9}{4} - 0 + C \quad C = \frac{9}{4}$$

$$y = -\frac{3}{4} \sqrt{9-4x^2} - \frac{5}{2} \sin^{-1}\left(\frac{2x}{3}\right) + \frac{9}{4} \quad \text{A1}$$

Note that a possible correct alternative answer is

$$y = -\frac{3}{4} \sqrt{9-4x^2} + \frac{5}{2} \cos^{-1}\left(\frac{2x}{3}\right) + \frac{9}{4} - \frac{5\pi}{4}$$

**Question 4**

- a. let  $P(z) = z^3 + pz^2 + qz + 15 = 0$  since  $p$  and  $q$  are real  
by the conjugate root theorem  $P(1-2i) = P(1+2i) = 0$   
so  $1+2i$  is also a root A1
- b. let  $\alpha = 1+2i$   $\beta = 1-2i$   
now  $\alpha + \beta = 2$  and  $\alpha\beta = 1-4i^2 = 5$   
so the quadratic  $z^2 - 2z + 5$  is a factor  
 $P(z) = z^3 + pz^2 + qz + 15 = (z^2 - 2z + 5)(z + c) = 0$  M1  
now  $5c = 15$  so that  $c = 3$  and expanding  $(z^2 - 2z + 5)(z + 3)$   
coefficient of  $z^2$ :  $p = 3 - 2 = 1$   
coefficient of  $z$ :  $q = 5 - 6 = -1$  A2  
and all the roots are  $z = 1 \pm 2i$  and  $z = -3$

**Question 5**

- a. **Method I** using addition theorems  $\frac{\pi}{12} = 15^\circ$   $15^\circ = 45^\circ - 30^\circ$
- $$\tan\left(\frac{\pi}{12}\right) = \tan\left(\frac{\pi}{4} - \frac{\pi}{6}\right) = \frac{\tan\left(\frac{\pi}{4}\right) - \tan\left(\frac{\pi}{6}\right)}{1 + \tan\left(\frac{\pi}{4}\right)\tan\left(\frac{\pi}{6}\right)}$$
- M1
- $$\tan\left(\frac{\pi}{12}\right) = \frac{1 - \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}} = \frac{\frac{\sqrt{3}-1}{\sqrt{3}}}{\frac{\sqrt{3}+1}{\sqrt{3}}} = \frac{\sqrt{3}-1}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$$
- M1
- $$\tan\left(\frac{\pi}{12}\right) = \frac{3 - 2\sqrt{3} + 1}{3 - 1} = \frac{4 - 2\sqrt{3}}{2}$$
- $$\tan\left(\frac{\pi}{12}\right) = 2 - \sqrt{3}$$
- A1

**Alternative Method II** using double angle formulae let  $A = \frac{\pi}{12}$   $2A = \frac{\pi}{6}$

$$\tan(2A) = \frac{2 \tan(A)}{1 - \tan^2(A)}$$

$$\frac{1}{\sqrt{3}} = \frac{2 \tan\left(\frac{\pi}{12}\right)}{1 - \tan^2\left(\frac{\pi}{12}\right)} \quad \text{M1}$$

$$1 - \tan^2\left(\frac{\pi}{12}\right) = 2\sqrt{3} \tan\left(\frac{\pi}{12}\right)$$

$$\tan^2\left(\frac{\pi}{12}\right) + 2\sqrt{3} \tan\left(\frac{\pi}{12}\right) - 1 = 0$$

$$\text{let } u = \tan\left(\frac{\pi}{12}\right) \quad u^2 + 2\sqrt{3}u - 1 = 0$$

$$\Delta = (2\sqrt{3})^2 + 4 = 16$$

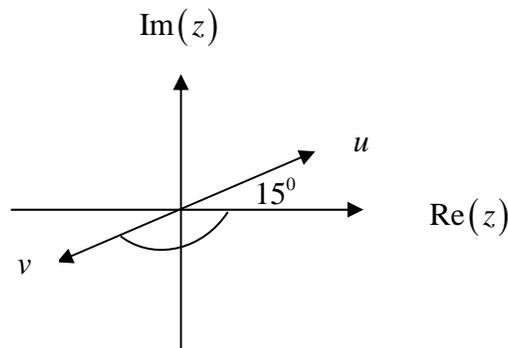
$$u = \tan\left(\frac{\pi}{12}\right) = \frac{-2\sqrt{3} \pm \sqrt{16}}{2} \quad \text{but } \tan\left(\frac{\pi}{12}\right) > 0 \quad \text{take the positive} \quad \text{M1}$$

$$\text{so that } \tan\left(\frac{\pi}{12}\right) = 2 - \sqrt{3} \quad \text{A1}$$

**b.** let  $u = 1 + (2 - \sqrt{3})i$  now  $\text{Arg}(u) = \tan^{-1}(2 - \sqrt{3}) = \frac{\pi}{12}$

let  $v = -1 + (\sqrt{3} - 2)i = -u = i^2 u$   $v$  is a rotation of  $180^\circ$  from  $u$ , so

$$\text{Arg}(v) = \text{Arg}(-1 + (\sqrt{3} - 2)i) = -\pi + \frac{\pi}{12} = -\frac{11\pi}{12} \quad (\text{or } -165^\circ) \quad \text{A1}$$



**Question 6**

a. Let  $y = \tan^{-1}\left(\sqrt{\frac{3}{x}}\right) = \tan^{-1}(u)$  where  $u = \sqrt{\frac{3}{x}} = \sqrt{3}x^{-\frac{1}{2}}$

$$\frac{dy}{du} = \frac{1}{1+u^2} \quad \frac{du}{dx} = -\frac{\sqrt{3}}{2}x^{-\frac{3}{2}} = \frac{-\sqrt{3}}{2\sqrt{x^3}} \quad \text{chain rule} \quad \text{M1}$$

$$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx} = \frac{-\sqrt{3}}{2\sqrt{x^3}\left(1+\frac{3}{x}\right)}$$

$$\frac{dy}{dx} = \frac{-\sqrt{3}}{2\sqrt{x^3}\left(\frac{x+3}{x}\right)} \quad \text{since } x > 0$$

$$\frac{dy}{dx} = \frac{-\sqrt{3}}{2\sqrt{x}(x+3)}$$

so shown  $\frac{d}{dx}\left(\tan^{-1}\left(\sqrt{\frac{3}{x}}\right)\right) = \frac{-\sqrt{3}}{2\sqrt{x}(x+3)}$  for  $x > 0$  A1

b.  $\int_1^9 \frac{1}{\sqrt{x^3+6x^2+9x}} dx = \int_1^9 \frac{1}{\sqrt{x(x^2+6x+9)}} dx =$

$$\int_1^9 \frac{1}{\sqrt{x(x+3)^2}} dx = \int_1^9 \frac{1}{\sqrt{x}(x+3)} dx \quad \text{since } x > 0$$

$$= -\frac{2}{\sqrt{3}} \left[ \tan^{-1}\left(\sqrt{\frac{3}{x}}\right) \right]_1^9 \quad \text{M1}$$

$$= -\frac{2}{\sqrt{3}} \left( \tan^{-1}\left(\frac{\sqrt{3}}{3}\right) - \tan^{-1}(\sqrt{3}) \right)$$

$$= -\frac{2\sqrt{3}}{3} \left( \frac{\pi}{6} - \frac{\pi}{3} \right)$$

$$= \frac{\sqrt{3}\pi}{9} \quad \text{A1}$$

**Question 7**

a.  $\underline{r}(t) = \frac{3}{2}(e^{2t} + e^{-2t})\underline{i} + \frac{5}{2}(e^{2t} - e^{-2t})\underline{j}$  vector equation,

the parametric equations are  $x = \frac{3}{2}(e^{2t} + e^{-2t})$  and  $y = \frac{5}{2}(e^{2t} - e^{-2t})$

now  $x^2 = \frac{9}{4}(e^{4t} + 2 + e^{-4t})$  and  $y^2 = \frac{25}{4}(e^{4t} - 2 + e^{-4t})$  A1

so that  $\frac{4x^2}{9} = (e^{4t} + 2 + e^{-4t})$  and  $\frac{4y^2}{25} = (e^{4t} - 2 + e^{-4t})$

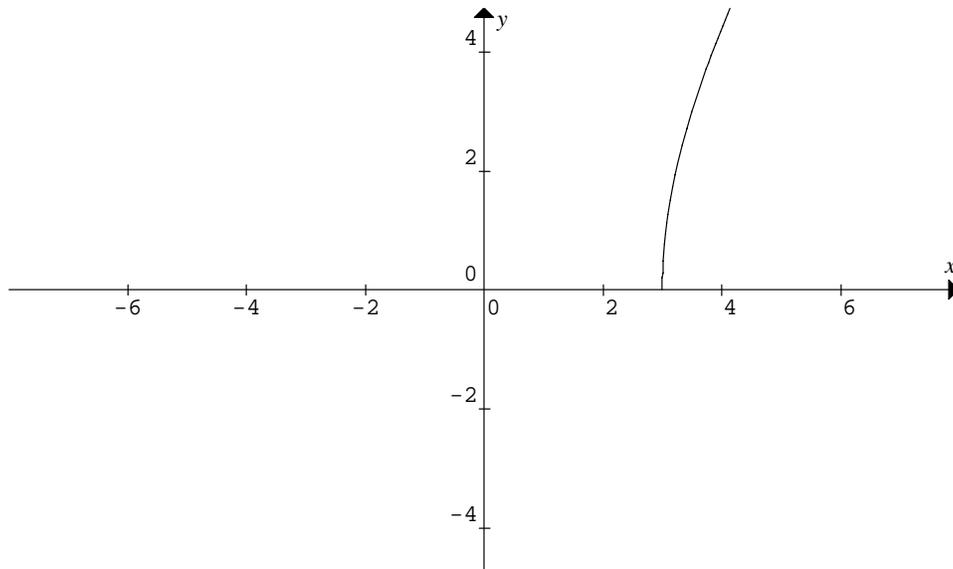
subtracting to eliminate  $t$  gives M1

$$\frac{4x^2}{9} - \frac{4y^2}{25} = 4$$

$$\frac{x^2}{9} - \frac{y^2}{25} = 1 \quad a^2 = 9 \quad b^2 = 25$$

since  $a > 0$  and  $b > 0$   $a = 3$   $b = 5$  A1

- b. since  $t \geq 0$  both  $x \geq 0$  and  $y \geq 0$ , the graph is not the whole hyperbola only the upper right branch, touching the  $x$ -axis at  $(3, 0)$  A1



**Question 8**

a.  $y = 6 \sin\left(\frac{\pi x}{2}\right)$

$$V = \pi \int_0^b 36 \sin^2\left(\frac{\pi x}{2}\right) dx \quad 0 < b < 4$$

$$V = 18\pi \int_0^b (1 - \cos(\pi x)) dx \quad \text{A1}$$

$$V = 18\pi \left[ x - \frac{1}{\pi} \sin(\pi x) \right]_0^b \quad \text{M1}$$

$$V = 18\pi \left[ b - \frac{1}{\pi} \sin(b\pi) - 0 \right]$$

$$V = 18(b\pi - \sin(b\pi)) \quad \text{A1}$$

b. if  $V = 18(b\pi - \sin(b\pi)) = 9(7\pi + 2)$

then  $b = \frac{7}{2}$  since  $\sin\left(\frac{7\pi}{2}\right) = -1$  A1

**Question 9**

a.  $y = \frac{4}{x^2 - 4x} = \frac{4}{x(x-4)}$

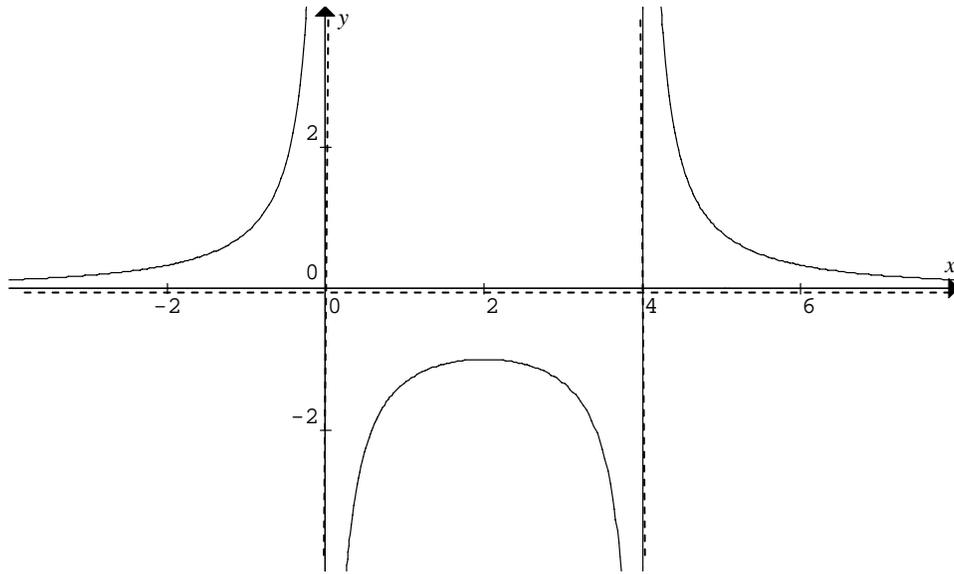
vertical asymptotes at  $x = 0$  ( the y-axis ) and  $x = 4$

horizontal asymptotes at  $y = 0$  ( the x-axis ) A1

the turning point is when  $2x - 4 = 0$  at  $x = 2$   $y(2) = \frac{4}{4-8} = -1$

maximum turning point at  $(2, -1)$

correct graph and turning point A1



b. the area  $= \int_1^2 \frac{4}{x^2 - 4x} dx = \int_1^2 \frac{4}{x(x-4)} dx$

by partial fractions  $\frac{4}{x^2 - 4x} = \frac{4}{x(x-4)} = \frac{A}{x} + \frac{B}{x-4}$  adding the partial fractions

$$= \frac{A(x-4) + Bx}{x(x-4)} = \frac{x(A+B) - 4A}{x^2 - 4x} \quad \text{M1}$$

(1)  $A+B=0$  and (2)  $-4A=4$  so that  $A=-1$  and  $B=-A$   $B=1$

the area  $A = \int_1^2 \left( \frac{1}{x-4} - \frac{1}{x} \right) dx$  but the area is under the  $x$ -axis, so  $A < 0$

$$A = \int_1^2 \left( \frac{1}{x} - \frac{1}{x-4} \right) dx \quad \text{now } A > 0 \quad \text{M1}$$

$$= \left[ \log_e(|x|) - \log_e(|x-4|) \right]_1^2$$

$$= \left[ \log_e \left( \left| \frac{x}{x-4} \right| \right) \right]_1^2 \quad \text{A1}$$

$$= \left( \log_e \left( \left| \frac{2}{-2} \right| \right) - \log_e \left( \left| \frac{1}{-3} \right| \right) \right)$$

$$= \log_e(3) \quad \text{A1}$$

**END OF SUGGESTED SOLUTIONS**