

Trial Examination 2006

VCE Specialist Mathematics Units 3 & 4

Written Examination 1

Question and Answer Booklet

Reading time: 15 minutes

Writing time: 1 hour

Student's Name: _____

Teacher's Name: _____

Structure of Booklet

Number of questions	Number of questions to be answered	Number of marks
9	9	40

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.

Students are NOT permitted to bring into the examination room: notes of any kind, a calculator, blank sheets of paper and/or white out liquid/tape.

Materials supplied

Question and answer booklet of 8 pages with a detachable sheet of miscellaneous formulas in the centrefold.

Working space is provided throughout the booklet.

Instructions

Write **your name** and your **teacher's name** in the space provided above on this page.

All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other electronic communication devices into the examination room.

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2006 VCE Specialist Mathematics Units 3 & 4 Written Examination 1.

Instructions

Answer all questions in the spaces provided.
 A decimal approximation will not be accepted if an exact answer is required to a question.
 In questions where more than one mark is available, appropriate working must be shown.
 Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

Question 1

Given that $z = 2 + 3i$ is a solution of the equation $z^3 - 6z^2 + 21z - 26 = 0$,

- a. write down another complex solution of the equation.

_____ 1 mark

- b. Hence find a quadratic factor of the left-hand side of the equation.

 _____ 2 marks

- c. Find the third solution.

 _____ 1 mark
 Total 4 marks

Question 2

A, B and C are three points on a plane such that $\vec{OA} = \underline{p}$, $\vec{OB} = 3\underline{q} - \underline{p}$ and $\vec{OC} = 9\underline{q} - 5\underline{p}$.

- a. Show that A, B and C are collinear.

 _____ 3 marks

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Formula Sheet

Directions to students

Detach this formula sheet during reading time.
This formula sheet is provided for your reference.

SPECIALIST MATHEMATICS FORMULAS

Mensuration

area of a trapezium: $\frac{1}{2}(a + b)h$

curved surface area of a cylinder: $2\pi rh$

volume of a cylinder: $\pi r^2 h$

volume of a cone: $\frac{1}{3}\pi r^2 h$

volume of a pyramid: $\frac{1}{3}Ah$

volume of a sphere: $\frac{4}{3}\pi r^3$

area of a triangle: $\frac{1}{2}bc \sin(A)$

sine rule: $\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$

cosine rule: $c^2 = a^2 + b^2 - 2ab \cos(C)$

Coordinate geometry

ellipse: $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

hyperbola: $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

Circular (trigonometric) functions

$\cos^2(x) + \sin^2(x) = 1$

$1 + \tan^2(x) = \sec^2(x)$

$\sin(x + y) = \sin(x)\cos(y) + \cos(x)\sin(y)$

$\cos(x + y) = \cos(x)\cos(y) - \sin(x)\sin(y)$

$\tan(x + y) = \frac{\tan(x) + \tan(y)}{1 - \tan(x)\tan(y)}$

$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$

$\sin(2x) = 2\sin(x)\cos(x)$

$\cot^2(x) + 1 = \operatorname{cosec}^2(x)$

$\sin(x - y) = \sin(x)\cos(y) - \cos(x)\sin(y)$

$\cos(x - y) = \cos(x)\cos(y) + \sin(x)\sin(y)$

$\tan(x - y) = \frac{\tan(x) - \tan(y)}{1 + \tan(x)\tan(y)}$

$\tan(2x) = \frac{2\tan(x)}{1 - \tan^2(x)}$

function	\sin^{-1}	\cos^{-1}	\tan^{-1}
domain	$[-1, 1]$	$[-1, 1]$	R
range	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Algebra (Complex numbers)

$z = x + yi = r(\cos(\theta) + i\sin(\theta)) = r\operatorname{cis}(\theta)$

$|z| = \sqrt{x^2 + y^2} = r$

$z_1 z_2 = r_1 r_2 \operatorname{cis}(\theta_1 + \theta_2)$

$z^n = r^n \operatorname{cis}(n\theta)$ (de Moivre's theorem)

$-\pi < \operatorname{Arg}(z) \leq \pi$

$\frac{z_1}{z_2} = \frac{r_1}{r_2} \operatorname{cis}(\theta_1 - \theta_2)$

Calculus

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\int x^n dx = \frac{1}{n+1}x^{n+1} + c, \quad n \neq -1$$

$$\frac{d}{dx}(e^{ax}) = ae^{ax}$$

$$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$$

$$\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$$

$$\int \frac{1}{x} dx = \log_e(x) + c, \quad \text{for } x > 0$$

$$\frac{d}{dx}(\sin(ax)) = a \cos(ax)$$

$$\int \sin(ax) dx = -\frac{1}{a} \cos(ax) + c$$

$$\frac{d}{dx}(\cos(ax)) = -a \sin(ax)$$

$$\int \cos(ax) dx = \frac{1}{a} \sin(ax) + c$$

$$\frac{d}{dx}(\tan(ax)) = a \sec^2(ax)$$

$$\int \sec^2(ax) dx = \frac{1}{a} \tan(ax) + c$$

$$\frac{d}{dx}(\sin^{-1}(x)) = \frac{1}{\sqrt{1-x^2}}$$

$$\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c, \quad a > 0$$

$$\frac{d}{dx}(\cos^{-1}(x)) = \frac{-1}{\sqrt{1-x^2}}$$

$$\int \frac{-1}{\sqrt{a^2-x^2}} dx = \cos^{-1}\left(\frac{x}{a}\right) + c, \quad a > 0$$

$$\frac{d}{dx}(\tan^{-1}(x)) = \frac{1}{1+x^2}$$

$$\int \frac{a}{a^2+x^2} dx = \tan^{-1}\left(\frac{x}{a}\right) + c$$

product rule:

$$\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$$

quotient rule:

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

chain rule:

$$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$$

Euler's method:

If $\frac{dy}{dx} = f(x)$, $x_0 = a$ and $(y_0 = b)$, then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x_n)$

acceleration:

$$a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v \frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

constant (uniform) acceleration: $v = u + at$ $s = ut + \frac{1}{2}at^2$ $v^2 = u^2 + 2as$ $s = \frac{1}{2}(u+v)t$

Vectors in two and three dimensions

$$\underline{r} = x\underline{i} + y\underline{j} + z\underline{k}$$

$$|\underline{r}| = \sqrt{x^2 + y^2 + z^2} = r$$

$$\underline{r}_1 \cdot \underline{r}_2 = r_1 r_2 \cos(\theta) = x_1 x_2 + y_1 y_2 + z_1 z_2$$

$$\dot{\underline{r}} = \frac{d\underline{r}}{dt} = \frac{dx}{dt}\underline{i} + \frac{dy}{dt}\underline{j} + \frac{dz}{dt}\underline{k}$$

Mechanics

momentum: $\underline{p} = m\underline{v}$

equation of motion: $\underline{R} = m\underline{a}$

sliding friction: $F \leq \mu N$

END OF FORMULA SHEET

c. Draw a diagram showing the forces acting on the passenger.

1 mark

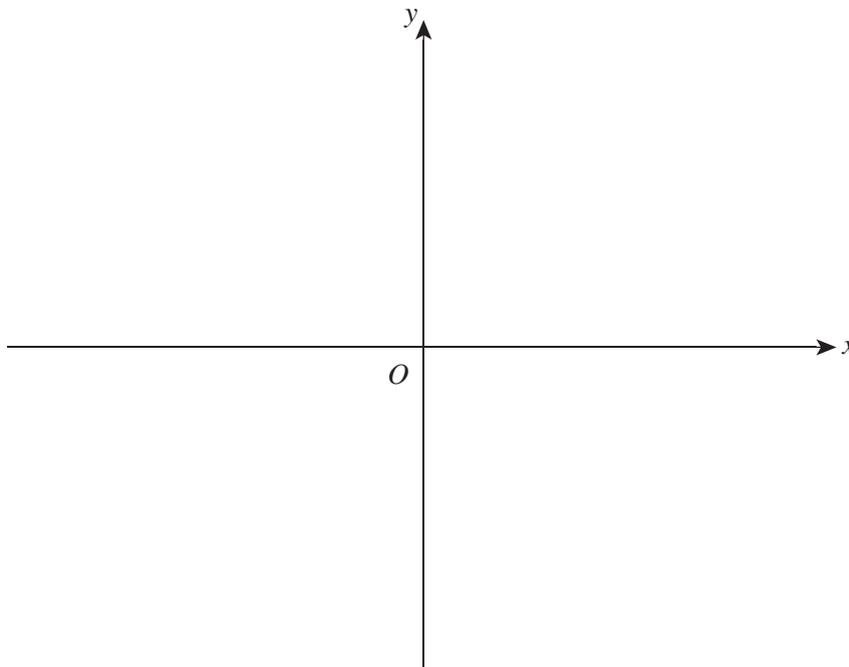
d. Find the force exerted by the floor of the lift on the passenger.

1 mark
Total 4 marks

Question 9

Consider the function with the rule $y = \frac{1}{x^2 + bx + c}$ where b and c are real valued constants.

a. Sketch the graph when $b = 0$ and $c = 1$ on the axes below, labelling any axis intercepts, stationary points and asymptotes.

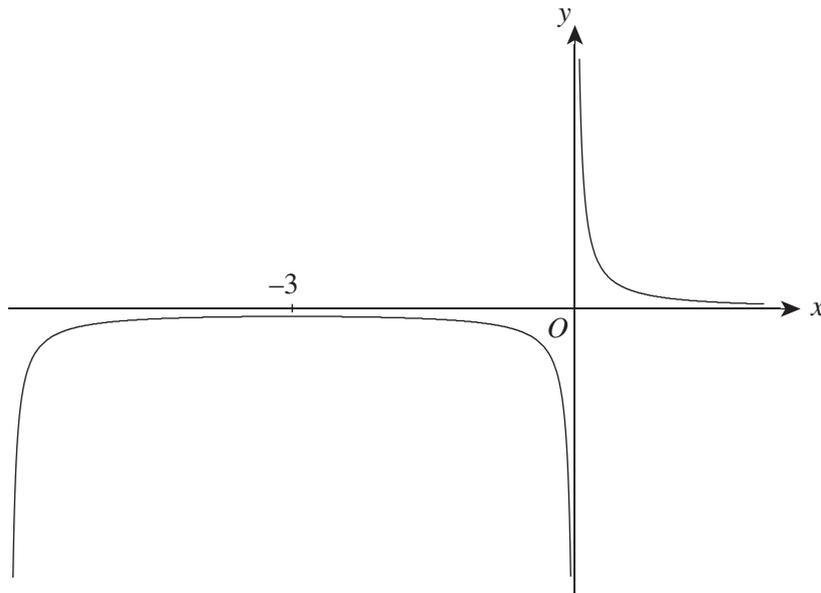


1 mark

b. What is the relationship between b and c so that the graph will have no vertical asymptotes?

1 mark

Part of the graph of the function f , with rule $f(x) = \frac{1}{x^2 + bx + c}$ is shown below. The function has a turning point when $x = -3$ and a vertical asymptote with equation $x = 0$.



c. Find $f'(x)$ and use it to show that $b = 6$.

2 marks

d. Find the value of c .

1 mark

- e. **Hence** find the exact area of the region enclosed by the graph of f , the x -axis and the lines $x = -2$ and $x = -1$.

3 marks
Total 8 marks

END OF QUESTION AND ANSWER BOOKLET