



THE SCHOOL FOR EXCELLENCE (TSFX) UNIT 4 MATHEMATICAL METHODS 2006

WRITTEN EXAMINATION 1

Reading Time: 15 minutes

Writing time: 1 hour

QUESTION AND ANSWER BOOKLET

Structure of Booklet

Number of questions	Number of questions to be answered	Number of marks
11	11	40

Students are permitted to bring into the examination rooms: pens, pencils, highlighters, erasers, sharpeners, rulers.

Students are **NOT** permitted to bring into the examination room: notes of any kind, a calculator, blank sheets of paper and/or white out liquid/tape.

Students are **NOT** permitted to bring mobile phones and/or any electronic communication devices into the examination room.

All written responses must be in English.

COMPLIMENTS OF THE SCHOOL FOR EXCELLENCE

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Instructions

- Answer **all** questions in the spaces provided.
- A decimal approximation will not be accepted if an **exact** answer is required to a question.
- In questions where more than 1 mark is available, appropriate working must be shown.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

QUESTION 1

Let $p(x) = 3x^4 + 4x^3 - 14x^2 - 20x - 5$.

- a. (i) Show that $3x + 1$ is a linear factor of $p(x)$.

- (ii) **Hence** express $p(x)$ in the form $(3x + 1)q(x)$ where $q(x)$ is a polynomial.

2 + 1 = 3 marks

QUESTION 2

A continuous random variable X has a probability distribution function given by

$$f(x) = \begin{cases} ax & \text{if } 0 < x < b \\ 0 & \text{otherwise} \end{cases}$$

- a. Show that $ab^2 = 2$.

- b. Find the mean value of X in terms of a and b .

- c. Find the values of a and b if the mean value of X is equal to 1.

1 + 1 + 2 marks

Total 4 marks

QUESTION 3

Find in exact form all real solutions to the equation $2^{3x} + 2^{2x} - 2^{-x} = 1$.

3 marks

QUESTION 4

For the function $f : (-\infty, 0] \rightarrow R$ with rule $f(t) = \frac{3t}{t^2 + 1}$, find the rule for the inverse function.

3 marks

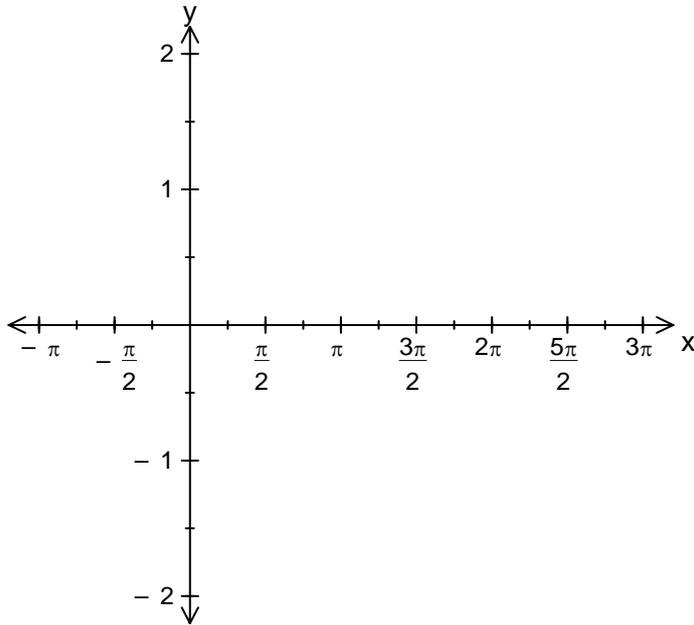
- c. Show that the curve $y = \frac{\cos^2 x}{x}$ also has stationary points with x -coordinates given by the solutions to the equation $2x \tan(x) = -1$.

3 + 1 + 1 marks

Total 5 marks

QUESTION 7

- a. Sketch the graph of the function $f : [-\pi, 3\pi] \rightarrow R$ with rule $f(x) = \cos\left(\frac{x}{2}\right)$ on the set of axes below.



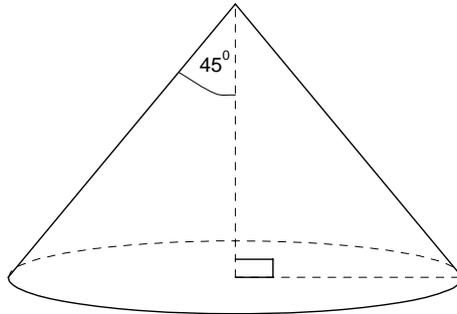
- b. On the same set of axes, sketch the graph of $y = |2f(x)|$.
- c. Hence find the exact area of the region bounded by the two curves.

1 + 1 + 2 marks

Total 4 marks

QUESTION 9

Grain is falling from the chute of a large storage container at a rate of $0.3 \text{ m}^3 / \text{min}$. The grain forms a heap on a flat horizontal floor in the shape of a right circular cone of semi-vertical angle 45° .



- a. Show that the exact height of the cone 30 minutes after the chute is first opened is $\frac{3}{\pi^{1/3}}$.

2 marks

QUESTION 11

The probability of obtaining a particular number of dots (the score), X , on the uppermost face of a biased die is shown in the following table.

x	1	2	3	4	5	6
$\Pr(X = x)$	p	$\frac{1}{7}$	$3p$	$3p$	$4p$	p

a. Find the exact value of p .

b. If a score greater than 2 was rolled, find the exact probability that the score was less than 5.

1 + 1 marks

Total 2 marks

END OF QUESTION AND ANSWER BOOKLET
