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Mathematical Methods

2006

Trial Examination 1

Instructions

Answer **all** questions. Do **not** use calculators.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working must be shown.

Unless otherwise indicated, the diagrams in this exam are **not** drawn to scale.

Question 1

For the function $f : (-1, \infty) \rightarrow \mathbb{R}$, $f(x) = \frac{e^{x+1}}{2}$,

a. find the rule of the inverse function f^{-1} .

b. find the domain of the inverse function f^{-1} .

2 + 1 = 3 marks

Question 2

a. Find $\frac{d}{dx}(2\sqrt{x} \cos(x))$.

b. Given $f'(x) = \frac{1}{(2-x)^2}$ and $f(3) = 3$, find the rule of f .

2 + 2 = 4 marks

Question 3

Find the smallest positive exact value (in terms of π) of the x -coordinate of the point where the function $y = \sqrt{3} \sin(x) + \cos(x)$ crosses the x -axis.

3 marks

Question 4

Given the function $f(x) = 2 \sin(3x)$, where $[0, 2\pi]$,

a. find the amplitude and period of the function g such that $g(x) = \frac{1}{3} f\left(\frac{x}{2}\right)$,

b. find the domain and range of the function h such that $h(x) = g(x) - \frac{2}{3}$.

2 + 2 = 4 marks

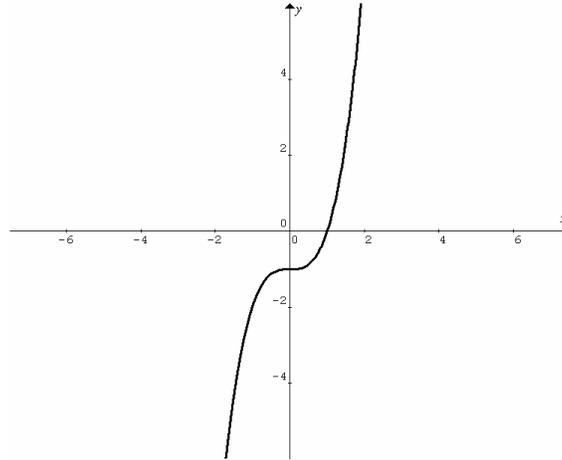
Question 5

The line $y = ax - 1$, $a > 0$, is a tangent to the curve $y = x^2$. Find the exact value of a .

3 marks

Question 6

The graph of $y = x^3 - 1$ is shown below.



- On the same set of axes (above) sketch the graph of $y = |x^3 - 1|$. Label all axis-intercepts.
- Hence find the exact area of the region bounded by the two curves and the y-axis.

- State the maximal domain of $\frac{dy}{dx}$ for $y = |x^3 - 1|$.

2 + 2 + 1 = 5 marks

Question 7

Given $u(x) = \log_e(x)$ and $f(x) = x^2 + 1$,

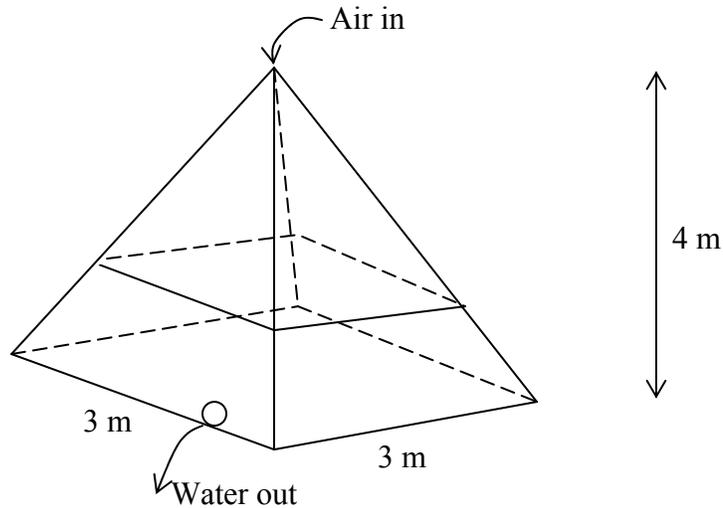
- write down the rule of $f(u(x))$,
- find the derivative of $f(u(x))$,
- hence find an anti-derivative of $\frac{\log_e x}{x}$.

1 + 1 + 1 = 3 marks

Question 8

An upright vessel in the shape of a right pyramid with a square base, side length 3 m and height 4 m, is initially filled with water. The water is pumped out from the vessel at a constant rate of 2 m^3 per minute. At what rate is the depth of the water falling when the depth is 1 m?

(Volume of pyramid = $\frac{1}{3} \times \text{base area} \times \text{height}$)



4 marks

Question 9

Random variable X is normally distributed with mean 5 and standard deviation 1.5.

a. Find $\Pr(X < 5)$.

b. Find $\Pr(X \geq 8)$.

1 + 1 = 2 marks

Question 10

Random variable X has a distribution with probability density function given by

$$f(x) = \begin{cases} 0.3 & \text{if } 0 \leq x < 2 \\ p & \text{if } 2 \leq x \leq 6 \\ 0 & \text{if } x < 0 \text{ or } x > 6 \end{cases}$$

a. Find the value of p .

b. Find $\Pr(-2 < X \leq 3)$.

2 + 2 = 4 marks

Question 11

A bag contains a red marble and two blue marbles. A marble is taken out of the bag at random, the colour is recorded and then returned to the bag. This is repeated for another two times. Set up a table of values to show the probability distribution of the random variable X : number of red marbles recorded in the three draws.

3 marks

Question 12

Maria drinks either tea or coffee. The probability of drinking a cup of tea after drinking a cup of coffee is 0.80, whereas the probability of drinking a cup of coffee after drinking a cup of tea is 0.90. Find the probability that her third cup is tea if she has started her day with a cup of coffee.

2 marks

End of Exam 1