

VCE Further Mathematics Units 3 & 4

Written Examination 1

Multiple-choice Question Booklet

Reading time: 15 minutes
Writing time: 1 hour 30 minutes

Student's Name: _____

Teacher's Name: _____

Structure of Booklet

Section	Number of questions	Number of questions to be answered	Number of modules	Number of modules to be answered	Marks
A – Core	13	13			13
B – Modules	54	27	6	3	27
					Total 40

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved graphics calculator or CAS and, if desired, one scientific calculator. Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white-out liquid/tape.

Materials supplied

Question booklet of 31 pages with a detachable formula sheet in the centrefold.
Answer sheet for multiple-choice questions.

Instructions

Detach the formula sheet from the centre of this booklet during reading time.

Please ensure that you write your **name** and your **teacher's name** in the space provided on this page and on the answer sheet for multiple-choice questions. Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

All written responses must be in English.

At the end of the examination

Place the answer sheet for multiple-choice questions inside the front cover of this booklet and hand them in.

Students are NOT permitted to bring mobile phones and/or any other electronic communication devices into the examination room.

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2006 VCE Further Mathematics Units 3 & 4 Written Examination 1.

SECTION A – DATA ANALYSIS – CORE MATERIAL**Instructions for Section A**

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.

Choose the response that is **correct** for the question.

A correct answer scores 1, an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No mark will be given if more than one answer is completed for any question.

The following information relates to Questions 1 and 2.

The percentage returns for several managed funds investing in international shares during the year 2005 were

12.6% 20.3% 11.2% 17.5% 9.8% 25.9% 15.3%

Question 1

The median investment return is

- A. 9.8%
- B. 11.2%
- C. 15.3%
- D. 20.3%
- E. 25.9%

Question 2

The range of investment returns is

- A. 9.8%
- B. 10.5%
- C. 14.7%
- D. 16.1%
- E. 25.9%

Question 3

A total of 1000 students sat an examination. The scores obtained by the students were normally distributed with a mean of 68. Approximately 95% of the results were between 46 and 90. From this we can conclude that the standard deviation is closest to

- A. 11
- B. 46
- C. 68
- D. 90
- E. 95

The following information relates to Questions 4 and 5.

A total of 50 individuals were asked how many different vehicles they had owned over a 10-year period. The results are displayed in the frequency table below.

Number of cars owned	Frequency
0	3
1	13
2	28
3	5
4	1

Question 4

What percentage of individuals owned at least two different cars in the last 10 years?

- A. 6%
- B. 32%
- C. 56%
- D. 68%
- E. 88%

Question 5

What is the mean number of cars owned per individual?

- A. 1
- B. 1.15
- C. 1.38
- D. 1.76
- E. 1.88

The following information relates to Questions 6, 7 and 8.

Several Year 12 mathematics classes of varying size (i.e. number of students in the group) sat a common exam. The class average was calculated for each group and the results are displayed in the table below.

Class size	Class average (%)
15	83
13	88
19	75
9	90
21	70
12	95
28	58
29	66
30	55
24	48

The equation for the least squares regression line is

$$\text{class average} = 109.35 - 1.827 \times \text{class size}$$

The coefficient of determination is 0.7471.

Question 6

Using the least squares regression line it can be estimated that the class average for a class of 20 students, to the nearest whole percentage, would be

- A. 70%
- B. 73%
- C. 75%
- D. 77%
- E. 95%

Question 7

From the equation it can be concluded that

- A. as class size increases, the class average decreases.
- B. as class size decreases, the class average decreases.
- C. as class size increases, the class average increases.
- D. as class size increases, the class average remains unchanged.
- E. as class size decreases, the class average remains unchanged.

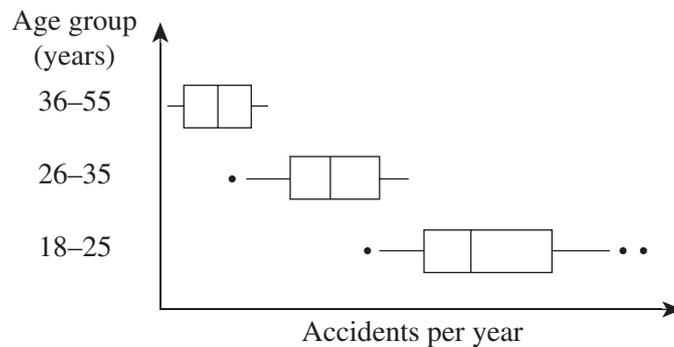
Question 8

The value of Pearson's product moment correlation coefficient, r , for this data (correct to two decimal places) is

- A. -0.86
- B. -0.75
- C. 0.56
- D. 0.75
- E. 0.86

Question 9

As part of an investigation, three samples were taken from the driving population. Each sample contained 100 drivers. One sample was taken from the 18 to 25 age group, one sample was taken from the 26 to 35 age group and the final sample was taken from the 36 to 55 age group. The parallel box plots below show the frequency of accidents per year for the three samples.

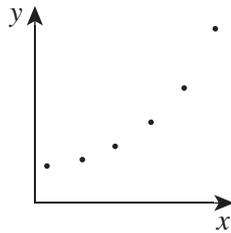


From the parallel box plots it can be concluded that, as the age of a driver increases, the number of accidents

- A. decreases on average and becomes less variable.
- B. decreases on average and becomes more variable.
- C. does not change on average but becomes more variable.
- D. increases on average and becomes less variable.
- E. increases on average and becomes more variable.

Question 10

The relationship between the two variables, y and x , as shown in the scatterplot below, is non-linear.



Which one of the following transformations, by itself, is most likely to linearise this data?

- A. A $\frac{1}{x}$ transformation.
- B. A $\frac{1}{y}$ transformation.
- C. An x^2 transformation.
- D. A $\log(x)$ transformation.
- E. A $\log(y)$ transformation.

The information below is for Question 11.

The sales figures (in thousands) for a company over a six month period were recorded as follows.

Month	Sales
1	26
2	34
3	40
4	45
5	51
6	59

When it was plotted on a scatterplot, the data showed that there was a strong linear association between the month and the associated sales figure.

Question 11

The three-median regression equation that would enable sales figures to be predicted from the month is closest to

- A. sales = $20.4 - 6.31 \times \text{month}$
- B. sales = $20.4 + 6.31 \times \text{month}$
- C. sales = $20.625 - 6.25 \times \text{month}$
- D. sales = $20.625 + 6.25 \times \text{month}$
- E. sales = $19.4 + 6.6 \times \text{month}$

Question 12

The data below gives the attendance figures recorded for the first round clash at the MCG between Carlton and Collingwood each year from 1990 to 2001.

Year	Attendance ($\times 1000$)
1990	78
1991	72
1992	66
1993	75
1994	83
1995	86
1996	76
1997	67
1998	74
1999	90
2000	79
2001	84

Using a **four** point moving average (mean) with centring, the smoothed value of the attendance in 1995 is

- A. 78
- B. 79
- C. 80
- D. 79 000
- E. 80 000

Question 13

A trend line is a straight line that can be used to represent an entire time series. Trend lines can be used for predicting the future values of a time series. These trend lines can be found in several ways.

Which of the following methods does **not** represent a way of finding a linear trend line?

- A. fitting by eye
- B. two-mean regression
- C. three-median regression
- D. de-seasonalisation
- E. least squares regression

END OF SECTION A – DATA ANALYSIS – CORE MATERIAL

SECTION B – MODULES

Instructions for Section B

Select **three** modules and answer **all** questions within the modules selected in pencil on the answer sheet provided for multiple-choice questions.

Show the modules you are answering by shading the matching boxes on your multiple-choice answer sheet.

Choose the response that is **correct** for the question.

A correct answer scores 1, an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No mark will be given if more than one answer is completed for any question.

Module	Page
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Module 1: Number patterns

Before answering these questions you must **shade** the “Number patterns” box on the answer sheet for multiple-choice questions.

Question 1

A certain arithmetic sequence has a first term of 8 and a third term of -2 .
The sum of the first four terms would be

- A. -7
- B. -5
- C. 2
- D. 4
- E. 5

Question 2

A certain geometric sequence has a sum to infinity of 10. The third term is $\frac{1}{4}$ of the first term.
The first term could be

- A. -10
- B. 0
- C. 10
- D. 15
- E. 30

Question 3

Steph has a sequence with the second, fourth and sixth terms being 16, 8 and 4 respectively.
Steph's sequence

- A. can be either arithmetic or geometric.
- B. can't be either arithmetic or geometric.
- C. must be arithmetic.
- D. must be geometric with the third term being $8\sqrt{2}$.
- E. can be geometric with the third term being $8\sqrt{2}$, but is not necessarily so.

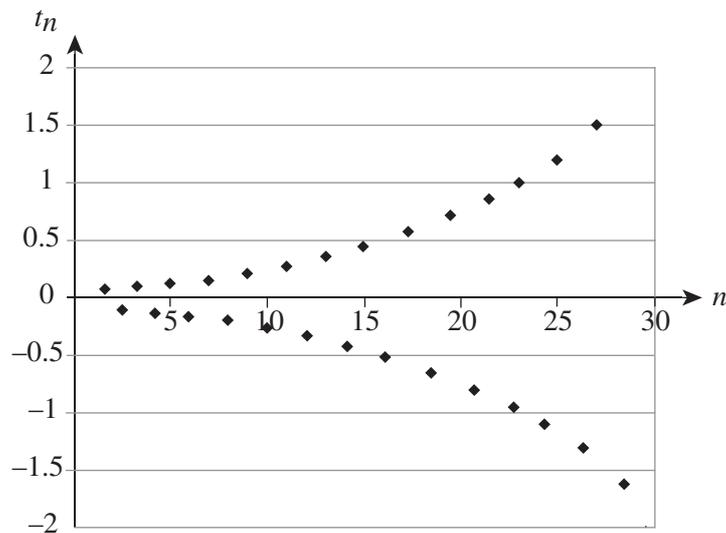
Question 4

Jacinta is aiming for 100%. Her first test for the year is only 36%. However, her aim is to halve the number of marks lost for each test she sits. If this succeeds, her mean mark for the five tests in first semester will be

- A. 13.95%
- B. 24.8%
- C. 75.2%
- D. 86.05%
- E. 124%

Question 5

The graph below shows the terms of a sequence t_n versus the number of the term n .



The sequence could be

- A. arithmetic, with a positive common difference.
- B. arithmetic, with a negative common difference.
- C. geometric, with a positive common ratio.
- D. geometric, with a negative common ratio.
- E. neither arithmetic nor geometric.

Question 6

In a certain card game, each player must bet \$1 more than the previous player in order to stay in the game. The first bet is always \$2. All of the bets are placed in the centre of the table. The total of these bets in a game in which n bets are placed would follow the difference equation

- A. $t_n = \frac{1}{2}n^2 + \frac{3n}{2}$
- B. $t_n = t_{n-1} + n + 1$
- C. $t_n = n + 1$
- D. $t_n = t_{n-1} + 1$
- E. $t_n = 2t_{n-1}$

Question 7

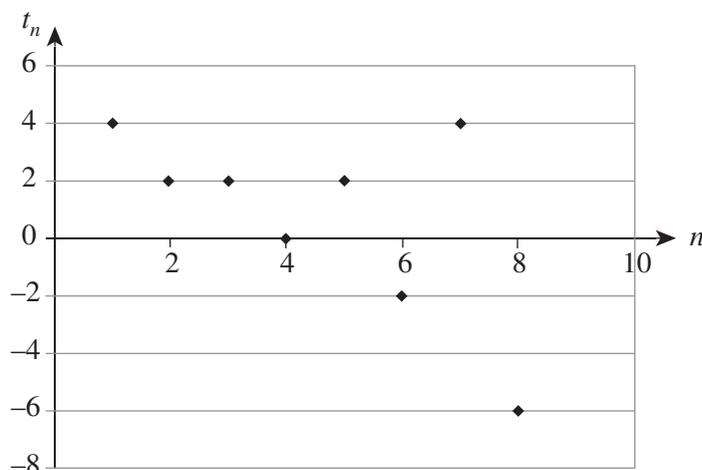
The sequence 6, -8, 20, -36, is modelled by the difference equation $t_{n+1} = b \times t_n + 4$, $t_0 = 6$.

The value of b is

- A. -4
- B. -3
- C. -2
- D. 1
- E. 2

Question 8

Michelle has a graph of a difference equation as shown below. She is unsure if the equation is first-order or higher-order.



From the graph we are able to conclude that

- A. the equation must be first-order since the values are decreasing.
- B. the equation must be first-order since terms 2 and 3 are the same.
- C. the equation could not be first-order since successive terms are diverging as n becomes large.
- D. the equation must not be first-order since terms 2 and 3 are the same but term 4 is different.
- E. the equation must be first-order since successive terms are diverging as n becomes larger.

Question 9

The solution of the difference equation $t_{n+1} = 3t_n - 2t_{n-1}$, $t_0 = 2$ and $t_1 = 5$, is

- A. $t_n = 2^n + 1$
- B. $t_n = 3(2^n) - 1$
- C. $t_n = n^2 + 1$
- D. $t_n = (n + 1)^2 + 1$
- E. $t_n = 3(2^n) - 2(2^{n-1})$

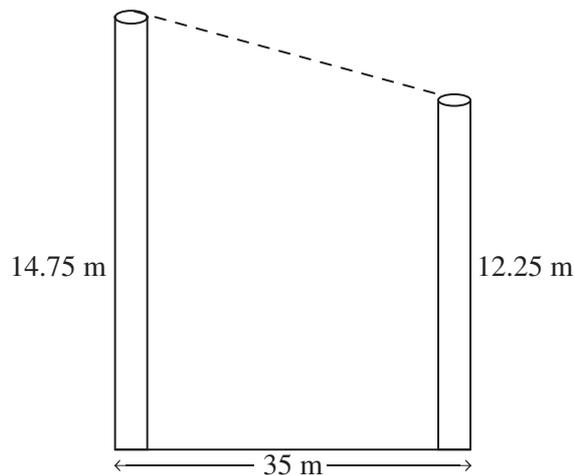
END OF MODULE 1

Module 2: Geometry and trigonometry

Before answering these questions you must **shade** the “Geometry and trigonometry” box on the answer sheet for multiple-choice questions.

Question 1

Questions 1 and 2 are based on the diagram below.



Two telephone poles are 35 metres apart. A cable is to connect the top of both poles. One pole has a height of 14.75 metres. The other pole has a height of 12.25 metres. The length of cable required to connect the top of both poles is closest to

- A. 2.50 m
- B. 34.95 m
- C. 35.08 m
- D. 35.09 m
- E. 37.50 m

Question 2

The angle of elevation of the top of the taller pole from the top of the smaller pole is closest to

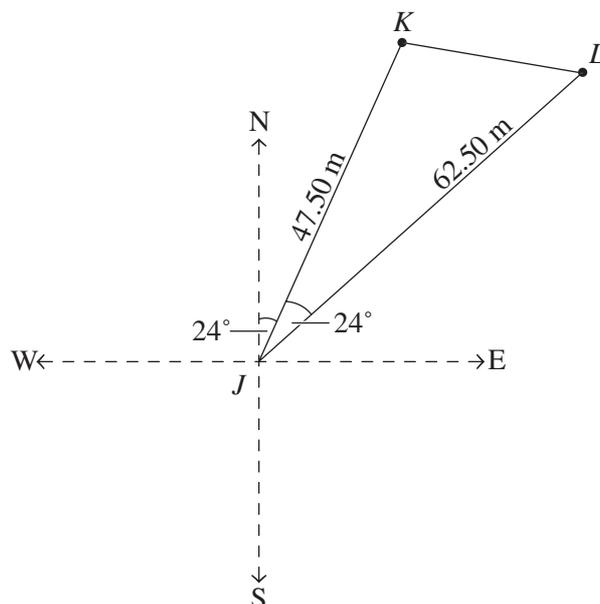
- A. 2°
- B. 2.5°
- C. 3°
- D. 4°
- E. 5°

Question 3

The radius of a large sphere is four times the radius of a small sphere. The ratio of the volume of the large sphere to the volume of the small sphere is

- A. 1:4
- B. 1:1
- C. 4:1
- D. 64:1
- E. 256:1

Questions 4, 5 and 6 refer to the following diagram.



Question 4

An area of lawn is to be created with artificial turf. Line JK is 47.50 metres long and has a true bearing of 24° . Line JL is 62.50 metres long and has a true bearing of 48° . The area of lawn required is closest to

- A. 502 m^2
- B. 604 m^2
- C. 1103 m^2
- D. 1356 m^2
- E. 1484 m^2

Question 5

The distance between points K and L is closest to

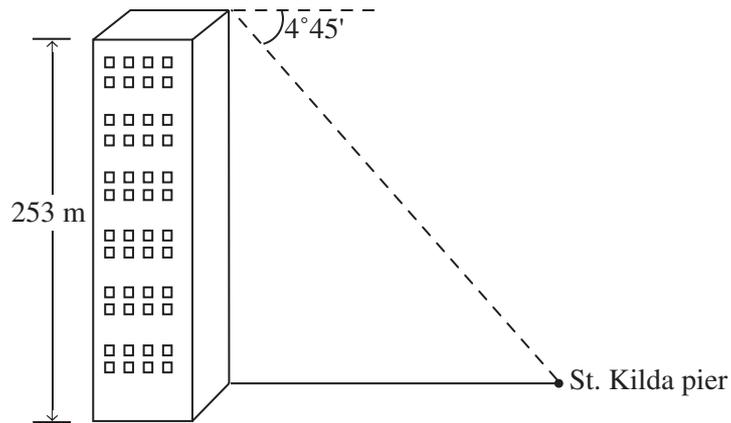
- A. 24.00 m
- B. 27.17 m
- C. 58.74 m
- D. 61.14 m
- E. 62.50 m

Question 6

The compass bearing of point J from point K is

- A. N 24° E
- B. S 24° E
- C. S 24° W
- D. N 42° E
- E. N 42° W

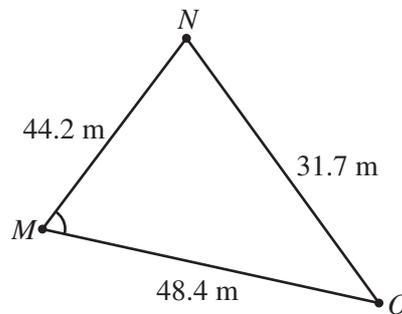
Question 7



The observation tower on the Rialto building in Melbourne is 253 metres above sea level. From the tower, the angle of depression to the kiosk on St. Kilda pier is $4^\circ 45'$. The horizontal distance from the St. Kilda pier to the base of the Rialto is closest to

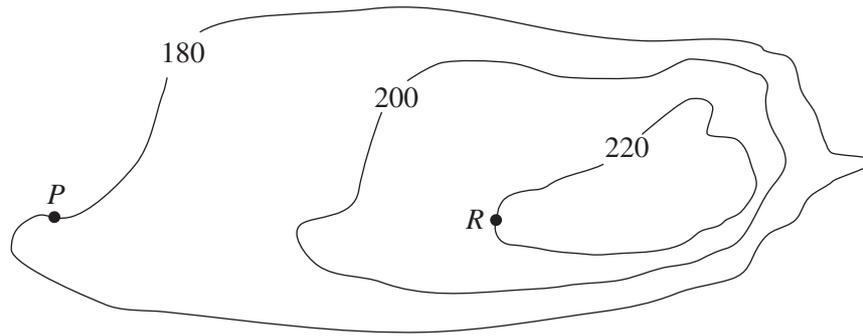
- A. 21 m
- B. 3251 m
- C. 3045 m
- D. 3929 m
- E. 4229 m

Question 8



The size of angle M is closest to

- A. 39°
- B. 40°
- C. 49°
- D. 50°
- E. 77°

Question 9

Points P and R are on the contour map above. The contour map has contours drawn at intervals of 20 metres. The average slope of the line PR is 15° . The horizontal distance between points P and R is closest to

- A. 10.4 m
- B. 10.7 m
- C. 38.6 m
- D. 149.3 m
- E. 159.4 m

END OF MODULE 2

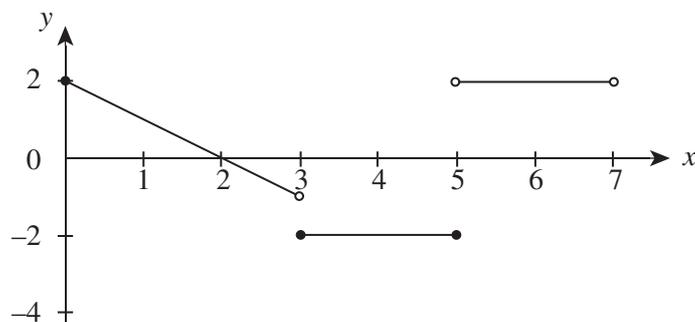
Module 3: Graphs and relations

Before answering these questions you must **shade** the “Graphs and relations” box on the answer sheet for multiple-choice questions.

Question 1

The line joining the points $P(3, -2)$ and $Q(-1, 0)$ would have the equation

- A. $y = \frac{1}{2}x - \frac{7}{2}$
 B. $y = 2x - 8$
 C. $y = 4 - 2x$
 D. $x + 2y = -1$
 E. $2x + y = 4$

Question 2

Which of the following rules would best describe the graph above?

- A. $y = \begin{cases} 2 - x & 0 \leq x < 3 \\ -2 & 3 \leq x \leq 5 \\ 3 & 5 < x < 7 \end{cases}$
 B. $y = \begin{cases} 2 - x & 0 < x < 3 \\ -2 & 3 < x < 5 \\ 2 & 5 \leq x \leq 7 \end{cases}$
 C. $y = \begin{cases} 2 - x & 0 < x < 3 \\ -2 & 3 < x < 5 \\ 2 & 5 < x \leq 7 \end{cases}$
 D. $y = \begin{cases} 2 - x & 0 \leq x < 3 \\ -2 & 3 \leq x \leq 5 \\ 2 & 5 \leq x \leq 7 \end{cases}$
 E. $y = \begin{cases} x + 2 & 0 \leq x \leq 3 \\ -2 & 3 \leq x \leq 5 \\ 2 & 5 < x < 7 \end{cases}$

Trial Examination 2006

VCE Further Mathematics Units 3 & 4

Written Examination 1

Formula Sheet

Detach this formula sheet during reading time.
This formula sheet is provided for your reference.

FURTHER MATHEMATICS FORMULAS**Core: Data analysis**

standardised score:
$$z = \frac{x - \bar{x}}{s_x}$$

least squares line:
$$y = a + bx \text{ where } b = r \frac{s_y}{s_x} \text{ and } a = \bar{y} - b\bar{x}$$

residual value:
$$\text{residual value} = \text{actual value} - \text{predicted value}$$

seasonal index:
$$\text{seasonal index} = \frac{\text{actual figure}}{\text{deseasonalised figure}}$$

Module 1: Number patterns

arithmetic series:
$$a + (a + d) + \dots + (a + (n - 1)d) = \frac{n}{2}[2a + (n - 1)d] = \frac{n}{2}(a + l)$$

geometric series:
$$a + ar + ar^2 + \dots + ar^{n-1} = \frac{a(1 - r^n)}{1 - r}, r \neq 1$$

infinite geometric series:
$$a + ar + ar^2 + ar^3 + \dots = \frac{a}{1 - r}, |r| < 1$$

Module 2: Geometry and trigonometry

area of a triangle:
$$\frac{1}{2}bc \sin A$$

Heron's formula:
$$A = \sqrt{s(s-a)(s-b)(s-c)} \text{ where } s = \frac{1}{2}(a + b + c)$$

circumference of a circle:
$$2\pi r$$

area of a circle:
$$\pi r^2$$

volume of a sphere:
$$\frac{4}{3}\pi r^3$$

surface area of a sphere:
$$4\pi r^2$$

volume of a cone:
$$\frac{1}{3}\pi r^2 h$$

volume of a cylinder:
$$\pi r^2 h$$

volume of a prism:
$$\text{area of base} \times \text{height}$$

volume of a pyramid:
$$\frac{1}{3} \text{ area of base} \times \text{height}$$

Pythagoras' theorem: $c^2 = a^2 + b^2$

sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

cosine rule: $c^2 = a^2 + b^2 - 2ab \cos C$

Module 3: Graphs and relations

Straight line graphs

gradient (slope): $m = \frac{y_2 - y_1}{x_2 - x_1}$

equation: $y = mx + c$

Module 4: Business-related mathematics

simple interest: $I = \frac{PrT}{100}$

compound interest: $A = PR^n$ where $R = 1 + \frac{r}{100}$

hire purchase: effective rate of interest $\approx \frac{2n}{n+1} \times \text{flat rate}$

Module 5: Networks and decision mathematics

Euler's formula: $v + f = e + 2$

Module 6: Matrices

determinant of a 2×2 matrix: $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$; $\det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

inverse of a 2×2 matrix: $A^{-1} = \frac{1}{\det A} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ where $\det A \neq 0$

END OF FORMULA SHEET

Question 3

For the linear relations

$$3x - 2y = 8$$

$$-6x + 4y = 8$$

it is **not** true that

- A. they are parallel.
- B. they both have positive gradients.
- C. they have the same x -intercept.
- D. at least one of them has a positive gradient.
- E. they do not intersect.

Question 4

A company has determined that costs associated with the operation of their swimwear factory are \$12 600 per year, which includes all costs except garment materials. They can sell each garment at \$11 more than the cost of materials to produce it. The minimum number of garments to be sold in order to make a profit is

- A. 1100 garments.
- B. 1145 garments.
- C. 1146 garments.
- D. 1150 garments.
- E. 12 589 garments.

Question 5

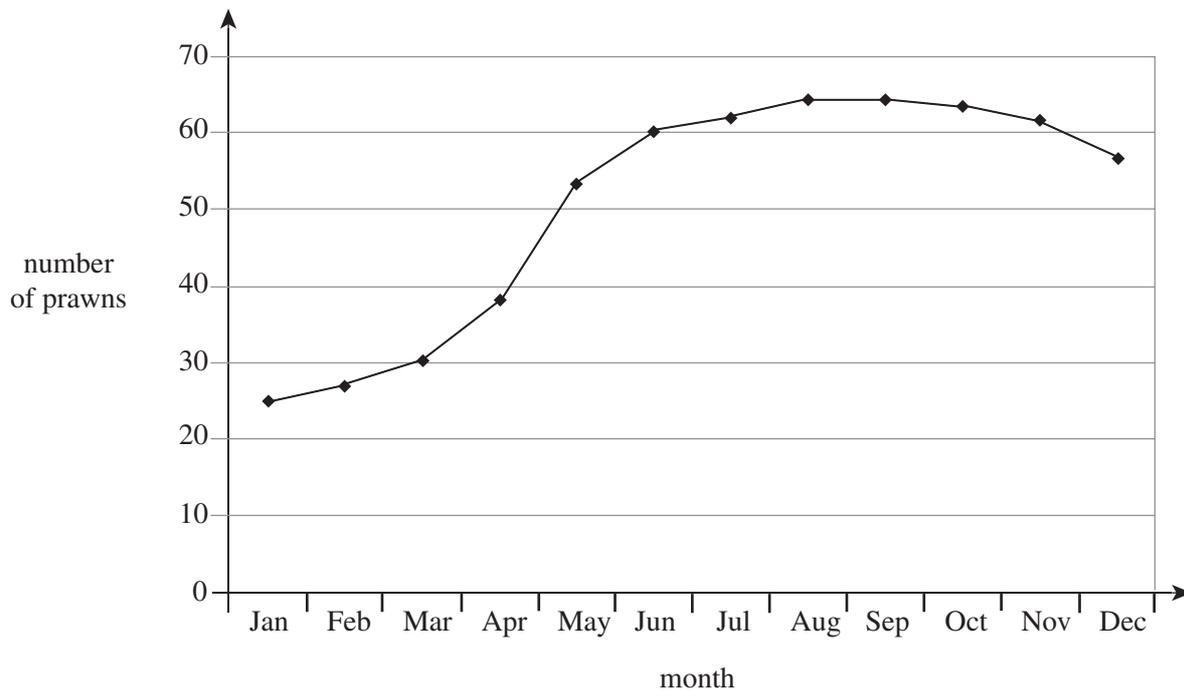
x	2	4	6	8
y	55	27.5	18.3	13.8

The data in the above table would best be plotted as a linear graph by plotting y against

- A. x
- B. x^2
- C. x^3
- D. $\frac{1}{x}$
- E. $\frac{1}{x^2}$

Question 6

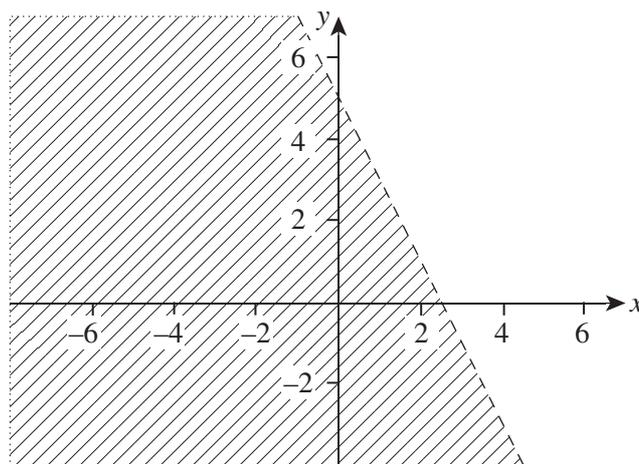
The graph below shows the number of prawns harvested at Ninkombah in 2005.



The greatest increase in harvested numbers occurred

- A. in August.
- B. in September.
- C. in April.
- D. in May.
- E. between April and May.

Question 7

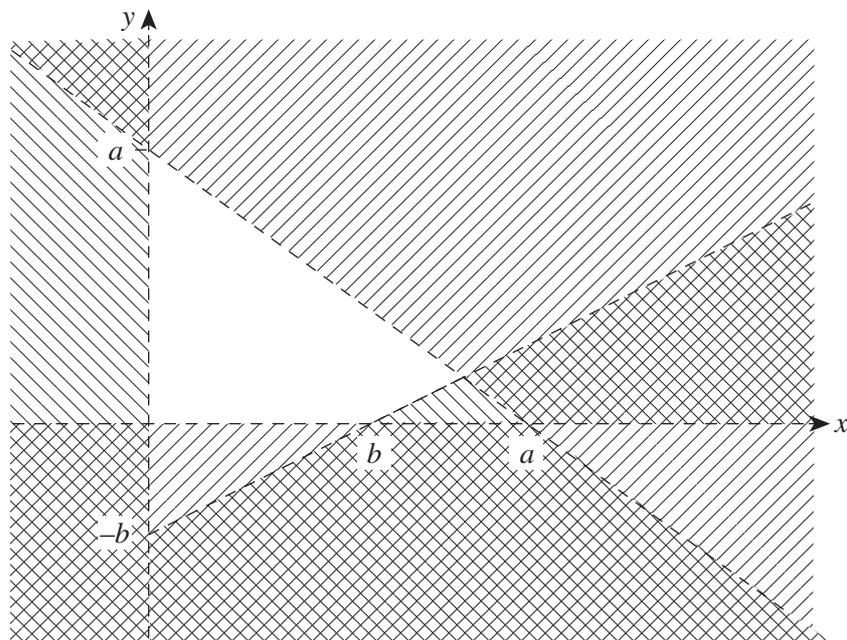


The graph shown (with the required region shaded and the border dashed) could be that of the inequation

- A. $3x + 5y = 15$
- B. $y < 5 - 2x$
- C. $y > 5 - 2x$
- D. $y \leq 5 - 2x$
- E. $y \geq 5 - 2x$

Question 8

The unshaded region has been formed by four separate constraints.



Which of these would be the equation of one of the constraints?

- A. $x < a - y$
- B. $x + y < b$
- C. $x + y > b$
- D. $x - y \leq b$
- E. $x - y > b$

Question 9

A combination of two different food supplements is to be used on a walking expedition through a jungle. Their characteristics are shown below (in standard units of each per kilogram).

	Riboflavin	Fat
<i>Supplement A</i>	5	5
<i>Supplement B</i>	3	4

The expedition leaders require that at least 45 units of riboflavin are consumed each day. The intake of fat from these supplements should not exceed 20 units. The set of constraints which could best apply is

- A.** $x \geq 0, y \geq 0$
 $5x + 3y \leq 45$
 $5x + 4y \leq 20$
- B.** $x \geq 0, y \geq 0$
 $5x + 3y \geq 45$
 $5x + 4y \leq 20$
- C.** $x \geq 0, y \geq 0$
 $5x + 5y \leq 45$
 $3x + 4y \leq 20$
- D.** $x \geq 0, y \geq 0$
 $5x + 5y \geq 45$
 $3x + 4y \leq 20$
- E.** $x \geq 0, y \geq 0$
 $5x + 3y \geq 45$
 $5x + 4y \geq 20$

END OF MODULE 3

Module 4: Business-related mathematics

Before answering these questions you must **shade** the “Business-related mathematics” box on the answer sheet for multiple-choice questions.

Question 1

Two banks pay simple interest on short-term deposits. Bank *A* pays 6% p.a. over four months and Bank *B* pays 6.5% p.a. for three months. If \$500 000 was invested in each account, the difference between the two banks' final payout figure would be

- A. \$0
- B. \$1875
- C. \$3333
- D. \$8125
- E. \$10 000

Question 2

A loan of \$5000 is taken over five years. The simple interest is calculated monthly. The interest bill on this loan is \$1125. The simple interest rate per year on this loan is

- A. 1.875%
- B. 3.75%
- C. 4%
- D. 4.5%
- E. 5%

Question 3

Date	Description	Debit	Credit	Balance
5/7	Transfer (A Brown)		\$200	
8/7	Salary		\$1700	
9/7	Check-45637	\$1450		\$706.50
29/7	ATM-Elwood	\$140		\$566.50

In the bank statement shown above, the minimum balance for the month is

- A. \$456.50
- B. \$2156.50
- C. \$706.50
- D. \$566.50
- E. \$256.50

Question 4

A hire-purchase contract specifies that there are to be monthly payments for three years. The flat rate of interest is 5.8% p.a. The effective interest rate for this contract is closest to

- A. 11.3%
- B. 10.7%
- C. 8.7%
- D. 5.8%
- E. 11.6%

Question 5

After five years, \$1500 has grown to \$2200 in an account where interest is compounded monthly. The annual interest rate is

- A. 6.4%
- B. 7.7%
- C. 8.0%
- D. 9.3%
- E. 38.9%

Question 6

A machine bought for \$28 500 is depreciated using the flat rate method. If its useful life is six years and its scrap value is \$3000 then the annual deduction for depreciation will be

- A. \$1491
- B. \$1895
- C. \$7250
- D. \$4250
- E. \$1366

Question 7

Tom repaid a reducing balance loan of \$35 000 in six years by quarterly repayments and with interest charged quarterly at 7.8% p.a. on the outstanding balance. The total amount of interest that he paid was closest to

- A. \$25 800
- B. \$2900
- C. \$2400
- D. \$9200
- E. \$10 200

Question 8

A loan of \$12 350 plus interest is to be repaid in 20 quarterly instalments of \$745. Interest at 7.3% per annum is calculated on the remaining balance each quarter. Which of the following statements is true?

- A. Most of the first instalment is used to pay the interest owing at that time.
- B. All of the interest is paid before the principal begins to reduce.
- C. The first instalment reduces the principal by a smaller amount than the second instalment.
- D. Every instalment reduces the principal by the same amount.
- E. The first instalment is not enough to pay for the interest owing at that time.

Question 9

Megan buys a new bed in a sale for \$1800, which is a 25% discount on the usual price. Later in the year she sees the same bed in a sale at a 30% discount, but discovers that the new discounted price is \$1890. This means that between the two sales, the usual price of the bed has increased. The percentage increase in the usual price of the bed between the two sales is closest to

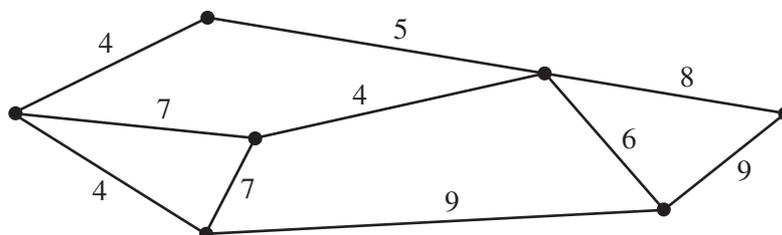
- A. 16.7%
- B. 15.9%
- C. 12.5%
- D. 11.1%
- E. 5.0%

END OF MODULE 4

Module 5: Networks and decision mathematics

Before answering these questions you must **shade** the “Networks and decision mathematics” box on the answer sheet for multiple-choice questions.

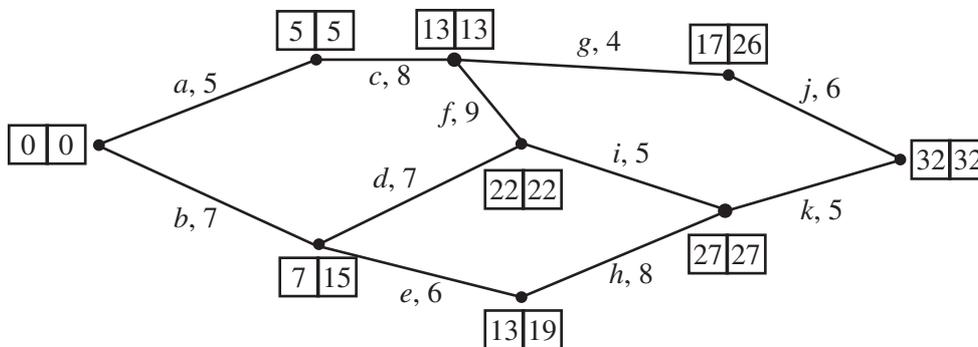
Question 1



The minimum spanning tree for the network above has a weight of

- A. 27
- B. 29
- C. 31
- D. 39
- E. 63

*Questions 2, 3 and 4 are all based on the network below.
The activities in the network are represented in minutes.*



Question 2

Which of the collection of edges are all on the critical path?

- A. a, c, e
- B. a, b, f
- C. b, i, k
- D. a, c, j
- E. c, f, k

Question 3

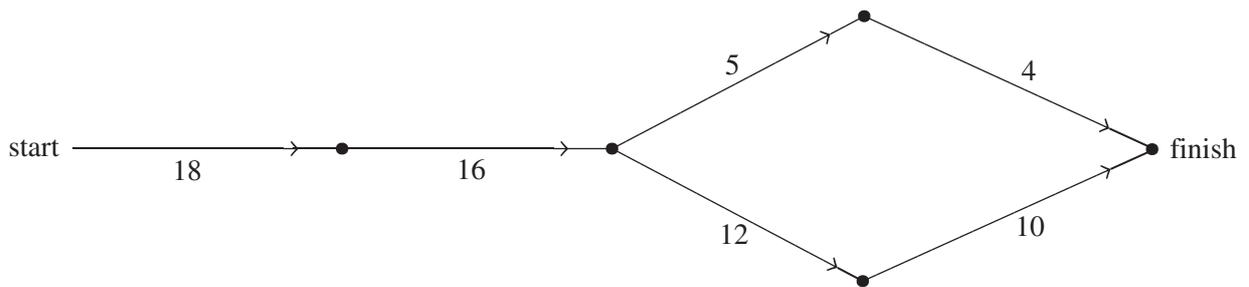
The 'slack' or 'float' time of activity 'd' is

- A. 7
- B. 8
- C. 10
- D. 15
- E. 22

Question 4

The time required to complete the entire project is

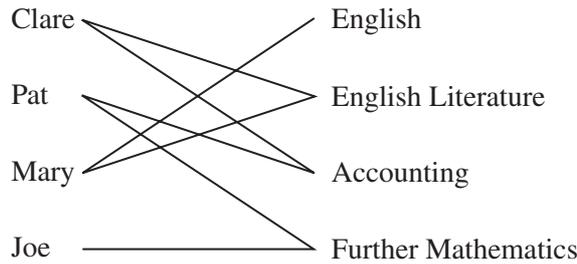
- A. 7
- B. 22
- C. 23
- D. 24
- E. 32

Question 5

Using the 'minimum-cut, maximum-flow' theorem, the maximum flow through the above network is

- A. 4
- B. 14
- C. 15
- D. 16
- E. 18

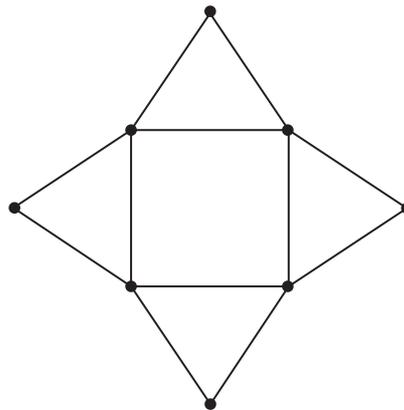
Question 6



The school principal at a secondary college needs to assign four teachers (Clare, Pat, Mary and Joe) to one class of English, one class of English Literature, one class of Accounting and one class of Further Mathematics. The bi-partite graph above describes the qualifications of the four teachers. Based on the bi-partite graph above, which statement is **not** true?

- A. Clare must teach English Literature and Pat must teach Accounting.
- B. Joe must teach Further Mathematics and Mary must teach English.
- C. Clare must teach English Literature and Mary must teach English.
- D. Joe must teach Further Mathematics and Mary must teach English Literature.
- E. Joe must teach Further Mathematics and Clare must teach English Literature.

Question 7



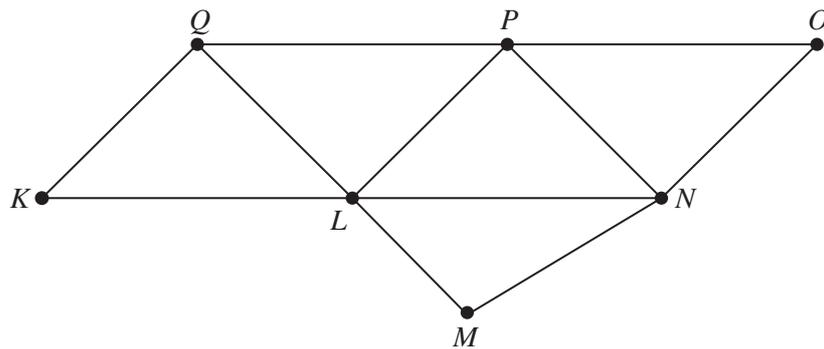
Which of the following descriptions of the number of vertices (v), edges (e) and faces (f), accurately represents the network above?

- A. $v = 8, e = 12$ and $f = 6$
- B. $v = 12, e = 8$ and $f = 6$
- C. $v = 8, e = 12$ and $f = 5$
- D. $v = 8, e = 8$ and $f = 6$
- E. $v = 8, e = 10$ and $f = 5$

Question 8

Two graphs are isomorphic if they

- A. have the same number of edges, faces and vertices.
- B. have the same number of edges, faces and vertices and are connected.
- C. are planar and connected.
- D. have the same number of edges and faces, but no loops or multiple edges.
- E. have the same number of edges and vertices and the same number of connections between them.

Question 9

Adding one edge to the network above can create an Euler Circuit. To do so, the added edge must connect the vertices

- A. O and P .
- B. L and Q .
- C. M and P .
- D. L and N .
- E. K and Q .

END OF MODULE 5

Module 6: Matrices

Before answering these questions you must **shade** the “Matrices” box on the answer sheet for multiple-choice questions.

Question 1

If $\begin{bmatrix} -2 & 0 & 1 \\ -1 & 3 & 2 \\ -3 & 4 & 5 \\ -5 & -4 & 7 \end{bmatrix}$, then $A_{4,2}$ is

- A. -4
- B. 2
- C. 4
- D. 5
- E. undefined

Question 2

If $P = \begin{bmatrix} -3 & 1 & 2 \\ -1 & -4 & 4 \\ 0 & 3 & -2 \end{bmatrix}$, $Q = \begin{bmatrix} 7 & -2 \\ 3 & 4 \\ 2 & 1 \end{bmatrix}$ and $R = \begin{bmatrix} -5 & 0 & 1 \\ -4 & 2 & 3 \end{bmatrix}$, which of the following matrix products

are undefined?

- A. PQ
- B. RP
- C. RQ
- D. $(PR)Q$
- E. $(QR)P$

Question 3

If $V = \begin{bmatrix} 5 & 2 \\ -1 & -3 \end{bmatrix}$, then the scalar product, $3V$, will be

- A. -39
- B. -13
- C. $\begin{bmatrix} 8 & 5 \\ 2 & 0 \end{bmatrix}$
- D. $\begin{bmatrix} 15 & 6 \\ -3 & -9 \end{bmatrix}$
- E. $\begin{bmatrix} -1 & 5 \\ -3 & -1 \end{bmatrix}$

Question 4

The equations

$$2x + 3y = 7$$

$$5x - 4y = 2$$

can be expressed in a matrix equation of the form $AX = B$, where $X = \begin{bmatrix} x \\ y \end{bmatrix}$.

Which of the following could be matrices A and B ?

A. $A = \begin{bmatrix} 2 & 3 \\ 5 & -4 \end{bmatrix}$, $B = \begin{bmatrix} 2 \\ 7 \end{bmatrix}$.

B. $A = \begin{bmatrix} 2 & 5 \\ 3 & -4 \end{bmatrix}$, $B = \begin{bmatrix} 7 \\ 2 \end{bmatrix}$.

C. $A = \begin{bmatrix} 2 & 5 \\ 3 & -4 \end{bmatrix}$, $B = \begin{bmatrix} 2 \\ 7 \end{bmatrix}$.

D. $A = \begin{bmatrix} 3 & 2 \\ -4 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 7 \\ 2 \end{bmatrix}$.

E. $A = \begin{bmatrix} 5 & -4 \\ 2 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 2 \\ 7 \end{bmatrix}$.

Question 5

The determinant of the matrix $\begin{bmatrix} 2 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & 1 & 2 \end{bmatrix}$ is

- A. -1
- B. 0
- C. 1
- D. 2
- E. 4

Question 6

A telecommunications company has two 3×3 matrices. Each matrix is organised so that the first, second and third columns represent 2004, 2005 and 2006 respectively and the first, second and third rows represent three different services. Matrix C contains the cost per minute for these services which are provided by a separate company responsible for installing the cabling. Matrix D gives the prices per minute at which these services are sold to customers. It should be assumed that no other costs exist, other than those indicated in these matrices.

Calculating $D - C$ would find

- A. the total profit per minute for these services in each of the three years.
- B. the total loss per minute for these services in each of the three years.
- C. the profit on each of these three services totalled across all three years.
- D. the profit on each of these three services in each of the three years.
- E. the percentage profit on each of these three services in each of the three years.

Question 7

A footwear company has expressed its 2006 prices in the form of a column vector, P , which is of the form

$$\begin{bmatrix} \text{price } x \\ \text{price } y \\ \text{price } z \end{bmatrix}. \text{ The prices for 2007 are to be found by the matrix product, } AP, \text{ where } A = \begin{bmatrix} 0.9 & 0 & 0 \\ 0 & 1.15 & 0 \\ 0 & 0 & 1.1 \end{bmatrix}.$$

Which of these is **not** a true result of this?

- A. Price y increases 15%.
- B. Price z increases 10%.
- C. All prices are altered.
- D. The price of each item in 2007 will depend only on the same item's price in 2006.
- E. Price x increases 90%.

The following information relates to questions 8 and 9.

Sue wants to find a transition matrix to determine the market share of three companies based only the

previous year. The three companies' share of the market will form a column vector $\begin{bmatrix} \text{company } a \\ \text{company } b \\ \text{company } c \end{bmatrix}$.

The transition matrix she chooses is $T = \begin{bmatrix} 0.5 & 0.3 & 0.4 \\ 0.3 & 0.4 & 0.2 \\ 0.2 & 0.3 & 0.4 \end{bmatrix}$.

Question 8

If the market share vector for 2005 is $V_0 = \begin{bmatrix} 0.4 \\ 0.5 \\ 0.1 \end{bmatrix}$, then Sue's prediction for the 2008 market share will be

A. $\begin{bmatrix} 0.42 & 0.39 & 0.42 \\ 0.31 & 0.31 & 0.28 \\ 0.27 & 0.3 & 0.3 \end{bmatrix}$

B. $\begin{bmatrix} 0.411 & 0.408 & 0.411 \\ 0.304 & 0.301 & 0.298 \\ 0.285 & 0.291 & 0.288 \end{bmatrix}$

C. $\begin{bmatrix} 0.39 \\ 0.34 \\ 0.27 \end{bmatrix}$

D. $\begin{bmatrix} 0.4098 \\ 0.3019 \\ 0.2883 \end{bmatrix}$

E. $\begin{bmatrix} 0.4110 \\ 0.3014 \\ 0.2877 \end{bmatrix}$

Question 9

Sue consults her friend Megan, who states that the transition matrix approaches a steady state when applied to the 2005 market share, but does not do so with other possible initial state vectors. Megan's statement is

- A.** true, because most possible initial state vectors will not result in a steady state.
- B.** true, because almost any possible initial state vector will result in a steady state.
- C.** false, because all possible vectors will result in a steady state, although the actual state will differ.
- D.** false, because all such vectors will result in a steady state of $\begin{bmatrix} 0.410959 \\ 0.30137 \\ 0.287671 \end{bmatrix}$.
- E.** false, because the transition matrix will not even approach a steady state for the 2005 market share vector.

END OF MULTIPLE-CHOICE QUESTION BOOKLET