

**Year 2006**  
**VCE**  
**Specialist Mathematics**  
**Solutions**  
**Trial Examination 1**



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**Question 1**

a.  $\underline{b} = -2\underline{i} + y\underline{j} + 4\underline{k}$

$$|\underline{b}| = \sqrt{(-2)^2 + y^2 + 4^2} = \sqrt{20 + y^2} = 5 \quad \text{squaring both sides} \quad \text{M1}$$

$$20 + y^2 = 25$$

$$y^2 = 5$$

$$y = \pm\sqrt{5} \quad \text{A1}$$

b.  $\cos(150^\circ) = -\frac{\sqrt{3}}{2} \quad \cos(\theta) = \frac{y}{|\underline{b}|}$

$$-\frac{\sqrt{3}}{2} = \frac{y}{\sqrt{20 + y^2}} \quad y < 0 \quad \text{M1}$$

$$-\sqrt{3}\sqrt{20 + y^2} = 2y \quad \text{squaring both sides}$$

$$3(20 + y^2) = 4y^2$$

$$60 + 3y^2 = 4y^2$$

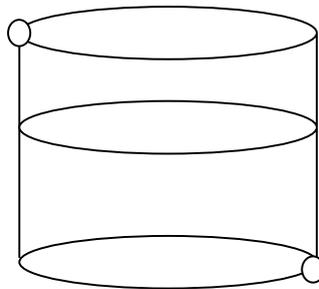
$$y^2 = 60 = 4 \times 15$$

$$y = -2\sqrt{15} \quad \text{since } y < 0 \quad \text{A1}$$

**Question 2**

a.

inflow 0.5  
kg/litre  
at 2 litre/min



outflow at 3 litre/min

Now  $\frac{dQ}{dt} = \text{inflow} - \text{outflow}$

and the volume  $V(t)$  of the tank at a time  $t$ .  $V(t) = 200 + (2 - 3)t = 200 - t$

$$\frac{dQ}{dt} = 2 \times 0.5 - \frac{3Q}{200 - t} = 1 - \frac{3Q}{200 - t} \quad \text{A1}$$

$$\text{b. } Q = \frac{1}{2}(200-t) + C(200-t)^n$$

$$\text{differentiating LHS } \frac{dQ}{dt} = -\frac{1}{2} - nC(200-t)^{n-1} \quad \text{M1}$$

$$\begin{aligned} \text{RHS } 1 - \frac{3Q}{200-t} &= 1 - \frac{3}{200-t} \left( \frac{1}{2}(200-t) + C(200-t)^n \right) \\ &= -\frac{1}{2} - 3C(200-t)^{n-1} \quad \text{therefore } n = 3 \quad \text{A1} \end{aligned}$$

**Question 3**

$$\text{a. } \text{Let } y = \cos^{-1} \left( \sqrt{\frac{3}{x}} \right) = \cos^{-1}(u) \quad \text{where } u = \sqrt{\frac{3}{x}} = \sqrt{3} x^{-\frac{1}{2}}$$

$$\frac{dy}{du} = \frac{-1}{\sqrt{1-u^2}} \quad \frac{du}{dx} = -\frac{\sqrt{3}}{2} x^{-\frac{3}{2}} = \frac{-\sqrt{3}}{2\sqrt{x^3}} \quad \text{M1}$$

$$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx} = \frac{\sqrt{3}}{2\sqrt{x^3} \sqrt{1-\frac{3}{x}}} = \frac{\sqrt{3}}{2\sqrt{x^3} \sqrt{\frac{x-3}{x}}}$$

$$\frac{dy}{dx} = \frac{\sqrt{3}}{2x\sqrt{x-3}} \quad \text{for } x > 3$$

$$\text{So shown } \frac{d}{dx} \left( \cos^{-1} \left( \sqrt{\frac{3}{x}} \right) \right) = \frac{\sqrt{3}}{2x\sqrt{x-3}} \quad \text{for } x > 3 \quad \text{A1}$$

$$\text{b. } \int_4^{12} \frac{1}{x\sqrt{x-3}} dx$$

$$= \frac{2}{\sqrt{3}} \left[ \cos^{-1} \left( \sqrt{\frac{3}{x}} \right) \right]_4^{12} \quad \text{M1}$$

$$= \frac{2}{\sqrt{3}} \left( \cos^{-1} \left( \frac{1}{2} \right) - \cos^{-1} \left( \frac{\sqrt{3}}{2} \right) \right)$$

$$= \frac{2\sqrt{3}}{3} \left( \frac{\pi}{3} - \frac{\pi}{6} \right)$$

$$= \frac{\sqrt{3}\pi}{9} \quad \text{A1}$$

## Question 4

a.  $y = \frac{x}{\sqrt{x-3}}$  using the quotient rule

$$u = x \quad v = \sqrt{x-3}$$

$$\frac{du}{dx} = 1 \quad \frac{dv}{dx} = \frac{1}{2}(x-3)^{-\frac{1}{2}} = \frac{1}{2\sqrt{x-3}}$$

M1

$$\frac{dy}{dx} = \frac{\sqrt{x-3} - \frac{x}{2\sqrt{x-3}}}{x-3} = \frac{2(x-3) - x}{2\sqrt{x-3}(x-3)}$$

$$\frac{dy}{dx} = \frac{x-6}{2\sqrt{(x-3)^3}} \quad \text{therefore} \quad a = \frac{1}{2} \quad b = -3$$

A1

b. the area  $A = \int_3^4 \frac{x dx}{\sqrt{x-3}}$  let  $u = x-3$   $\frac{du}{dx} = 1$   $x = u+3$

terminals when  $x=4$   $u=1$  and when  $x=3$   $u=0$

$$A = \int_0^1 \frac{u+3}{\sqrt{u}} du = \int_0^1 \left( u^{\frac{1}{2}} + 3u^{-\frac{1}{2}} \right) du$$

M1

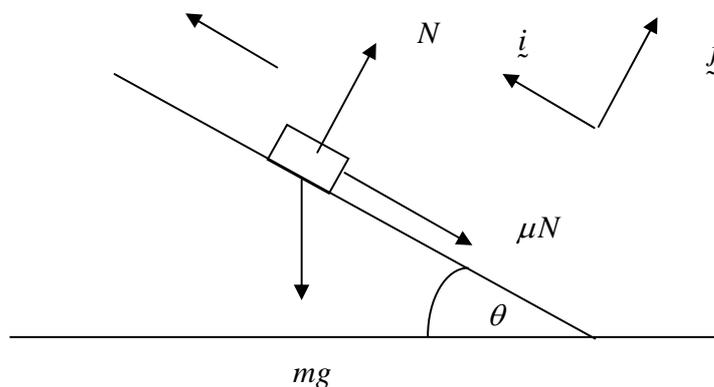
$$A = \left[ \frac{2}{3}u^{\frac{3}{2}} + 6u^{\frac{1}{2}} \right]_0^1 = \left( \frac{2}{3}(1) + 6\sqrt{1} - 0 \right)$$

$$A = 6\frac{2}{3}$$

A1

## Question 5

a.



A1

b. resolving up parallel to the slide in the  $\underline{i}$  direction.

$$(1) ma = -mg \sin(\theta) - \mu N$$

resolving perpendicular to the slide in the  $\underline{j}$  direction.

$$(2) N - mg \cos(\theta) = 0 \quad (2) \Rightarrow N = mg \cos(\theta) \text{ into (1)} \quad \text{M1}$$

$$ma = -mg \sin(\theta) - \mu mg \cos(\theta) = -mg (\sin(\theta) + \mu \cos(\theta))$$

$$a = -g (\sin(\theta) + \mu \cos(\theta)) \quad \text{A1}$$

Using constant acceleration formulae  $v^2 = u^2 + 2as$  with  $u = U$   $v = 0$   $s = D = ?$

$$0 = U^2 - 2g (\sin(\theta) + \mu \cos(\theta)) D$$

$$D = \frac{U^2}{2g (\sin(\theta) + \mu \cos(\theta))} \quad \text{A1}$$

### Question 6

$$V = \pi \int_a^b (y_1^2 - y_2^2) dx$$

$$V = \pi \int_0^{\frac{\pi}{6}} (\cos^2(2x) - \sin^2(x)) dx \quad \text{M1}$$

$$V = \pi \int_0^{\frac{\pi}{6}} \left( \left( \frac{1}{2} + \frac{1}{2} \cos(4x) \right) - \left( \frac{1}{2} - \frac{1}{2} \cos(2x) \right) \right) dx \quad \text{M1}$$

$$V = \frac{\pi}{2} \int_0^{\frac{\pi}{6}} (\cos(4x) + \cos(2x)) dx \quad \text{A1}$$

$$V = \frac{\pi}{2} \left[ \frac{1}{4} \sin(4x) + \frac{1}{2} \sin(2x) \right]_0^{\frac{\pi}{6}} = \frac{\pi}{2} \left[ \left( \frac{1}{4} \sin\left(\frac{2\pi}{3}\right) + \frac{1}{2} \sin\left(\frac{\pi}{3}\right) \right) - 0 \right]$$

$$V = \frac{\pi}{2} \left( \frac{\sqrt{3}}{8} + \frac{\sqrt{3}}{4} \right)$$

$$V = \frac{3\pi\sqrt{3}}{16} \quad \text{A1}$$

**Question 7**

**a.**  $z^4 + z^2 - 12 = 0$   
 $(z^2 - 3)(z^2 + 4) = 0$  M1

$$(z^2 - (\sqrt{3})^2)(z^2 - 4i^2) = 0$$

$$(z + \sqrt{3})(z - \sqrt{3})(z - 2i)(z + 2i) = 0$$

$$z = \pm\sqrt{3} \quad z = \pm 2i$$
 A1

**b.**  $z^2 = (a + bi)^2 = -1 - 4\sqrt{3}i$  where  $a, b \in R$

$$z^2 = (a^2 + 2abi + b^2i^2) = (a^2 - b^2) + 2abi = -1 - 4\sqrt{3}i$$

equating real and imaginary parts

real (1)  $a^2 - b^2 = -1$  M1

imaginary (2)  $2ab = -4\sqrt{3}$  from (2)  $b = -\frac{2\sqrt{3}}{a}$  substitute into (1)

$$a^2 - \left(\frac{-2\sqrt{3}}{a}\right)^2 = -1$$
 M1

$$a^2 - \frac{12}{a^2} + 1 = 0 \quad \text{multiply by } a^2$$

$$a^4 + a^2 - 12 = 0 \quad \text{from i. since } a \text{ is real}$$

$$a = \pm\sqrt{3} \quad \text{and } b = \mp 2 \quad \text{so } (\pm(\sqrt{3} - 2i))^2 = -1 - 4\sqrt{3}i$$

therefore if  $z^2 = -1 - 4\sqrt{3}i$  then  $z = \pm(\sqrt{3} - 2i)$  A1

**c.**  $z^2 + \sqrt{3}z + (1 + \sqrt{3}i) = 0$

using the quadratic formulae with  $a = 1$   $b = \sqrt{3}$   $c = 1 + \sqrt{3}i$

$$\Delta = b^2 - 4ac = 3 - 4(1 + \sqrt{3}i) = -1 - 4\sqrt{3}i$$
 A1

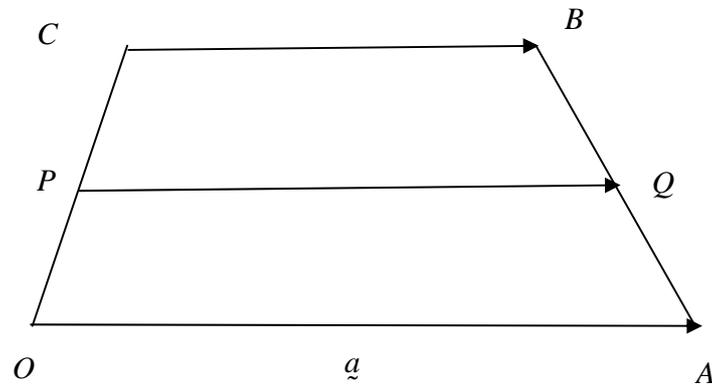
so  $\sqrt{\Delta} = \sqrt{(-1 - 4\sqrt{3}i)} = \sqrt{3} - 2i$

$$z = \frac{-b \pm \sqrt{\Delta}}{2a}$$

$$z = \frac{-\sqrt{3} \pm (\sqrt{3} - 2i)}{2}$$
 M1

$$z = -i \quad \text{and} \quad -\sqrt{3} + i$$
 A1

## Question 8



a.  $\overrightarrow{OA} = \underline{a} = \lambda \overrightarrow{CB} \quad \overrightarrow{CB} = \frac{1}{\lambda} \underline{a}$

since P is the midpoint of  $\overrightarrow{OC}$   $\overrightarrow{OP} = \overrightarrow{PC} = \frac{1}{2} \overrightarrow{OC}$

since Q is the midpoint of  $\overrightarrow{AB}$   $\overrightarrow{AQ} = \overrightarrow{QB} = \frac{1}{2} \overrightarrow{AB}$

$$\overrightarrow{PQ} = \overrightarrow{PC} + \overrightarrow{CB} + \overrightarrow{BQ} \quad (1)$$

$$\overrightarrow{PQ} = \overrightarrow{PO} + \overrightarrow{OA} + \overrightarrow{AQ} \quad (2)$$

M1

adding these two equations gives

$$2\overrightarrow{PQ} = \overrightarrow{PC} + \overrightarrow{PO} + \overrightarrow{CB} + \overrightarrow{OA} + \overrightarrow{BQ} + \overrightarrow{AQ}$$

M1

$$2\overrightarrow{PQ} = \overrightarrow{CB} + \overrightarrow{OA}$$

$$2\overrightarrow{PQ} = \left(\frac{1}{\lambda} + 1\right) \underline{a}$$

$$\overrightarrow{PQ} = \left(\frac{\lambda + 1}{2\lambda}\right) \underline{a}$$

A1

b. the length of  $\overrightarrow{PQ} = |\overrightarrow{PQ}|$  is equal to  $\frac{\lambda + 1}{2\lambda}$  times the length of  $\underline{a}$

so the ratio of the length is  $\frac{\lambda + 1}{2\lambda}$

A1

**Question 9**

- a. vertical asymptotes at  $x = -4$  and  $x = -2$  so the denominator is  
 $(x+4)(x+2) = x^2 + 6x + 8$  therefore  $b = 6$   $c = 8$  A1  
 the turning point is  $(-3, -2)$

$$\text{when } x = -3 \quad y = -2 \quad \text{so} \quad -2 = \frac{a}{(-3)^2 - 18 + 8} = \frac{a}{-1}$$

$$\text{therefore } a = 2 \quad \text{A1}$$

b. the area  $= \int_0^2 \frac{2}{x^2 + 6x + 8} dx = \int_0^2 \frac{2}{(x+4)(x+2)} dx$

$$\text{by partial fractions } \frac{2}{x^2 + 6x + 8} = \frac{2}{(x+4)(x+2)} = \frac{A}{x+4} + \frac{B}{x+2}$$

$$\frac{A}{x+4} + \frac{B}{x+2} \quad \text{Now adding the partial fractions}$$

$$= \frac{A(x+2) + B(x+4)}{(x+4)(x+2)} = \frac{x(A+B) + 2A + 4B}{x^2 + 6x + 8}$$

$$(1) \quad A + B = 0 \quad \text{and} \quad (2) \quad 2A + 4B = 2 \quad \text{from (1) } A = -B \quad \text{into (2)}$$

$$2B = 2 \quad B = 1 \quad \text{and} \quad A = -1 \quad \text{A1}$$

$$\text{area} = \int_0^2 \left( \frac{1}{x+2} - \frac{1}{x+4} \right) dx$$

$$= \left[ \log_e \left( \frac{x+2}{x+4} \right) \right]_0^2 = \left( \log_e \left( \frac{4}{6} \right) - \log_e \left( \frac{2}{4} \right) \right) \quad \text{A1}$$

$$= \log_e \left( \frac{4}{3} \right) \quad \text{therefore } p = \frac{4}{3} \quad \text{A1}$$

**END OF SUGGESTED SOLUTIONS**