Insight Publications

INSIGHT

Trial Exam Paper

2006

SPECIALIST MATHEMATICS

Written examination 2

STUDENT NAME:

QUESTION AND ANSWER BOOK

Reading time: 15 minutes
Writing time: 2 hours

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
1	22	22	22
2	5	5	58
		Total	80

- Students are permitted to bring the following items into the examination: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, once bound reference, one approved graphics calculator or CAS (memory DOES NOT need to be cleared) and, if desired, one scientific calculator.
- Students are NOT permitted to bring sheets of paper or white out liquid/tape into the examination.

Materials provided
- The question and answer book of 27 pages with a separate sheet of miscellaneous formulas.
- Answer sheet for multiple-choice questions

Instructions
- Write your **name** in the box provided and on the multiple-choice answer sheet.
- Remove the formula sheet during reading time.
- You must answer the questions in English.

At the end of the exam
- Place the multiple-choice answer sheet inside the front cover of this book.

Students are NOT permitted to bring mobile phones or any other electronic devices into the examination.

SECTION 1

Instructions for Section 1

Answer **all** questions in pencil on the multiple-choice answer sheet provided.
Choose the response that is **correct** for the question.
One mark will be awarded for a correct answer; no marks will be awarded for an incorrect answer.
Marks **are not** deducted for incorrect answers.
No marks will be awarded if more than one answer is completed for any question.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$

Question 1

If $2i$ is a solution of the equation $z^3 - 5z^2 + 4z - mi = 0$, then the value of m will be

A. $-2i$

B. $-20i$

C. -20

D. 20

E. $20i$

Question 2

If $z = -1 + \sqrt{3}$, then Arg (z^2) equals

A. $-\frac{2\pi}{3}$

B. $-\frac{\pi}{3}$

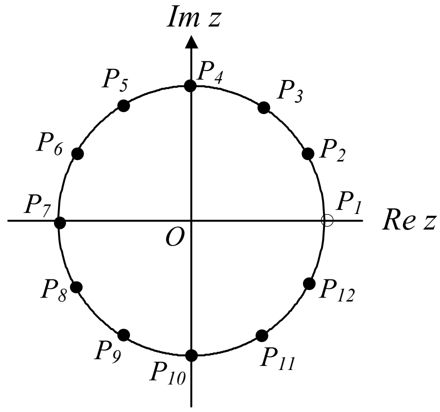
C. $\frac{\pi}{3}$

D. $\frac{2\pi}{3}$

E. $\frac{4\pi}{3}$

Question 3

Points P_1 to P_{12} are twelve equally spaced points around the circumference of a circle.



Point P_3 represents the complex number $z = a + ib$.

The complex number $i^{11}\bar{z}$ is represented by point

A. P_2

B. P_5

C. P_8

D. P_9

E. P_{11}

Question 4

The range of the function $f(x) = \cos^{-1}(x - \pi) - 1$ is

A. $[\pi - 1,\ \pi + 1]$

B. $[-1,\ \pi - 1]$

C. $[0,\ \pi]$

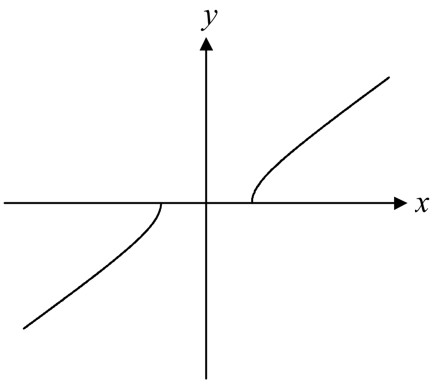
D. $[-2,\ 0]$

E. $[-1,\ 1]$

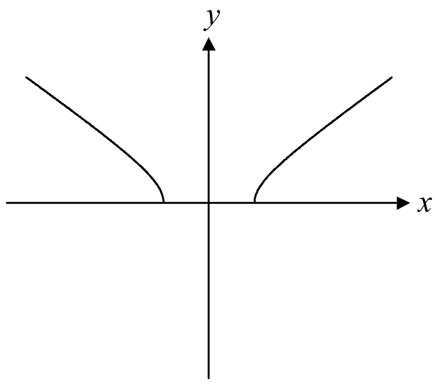
Question 5

A graph of the curve specified by the parametric equations $x = \sec(t)$, $y = \tan(t)$ where $t \in [0, \pi]$ could be

A.



B.



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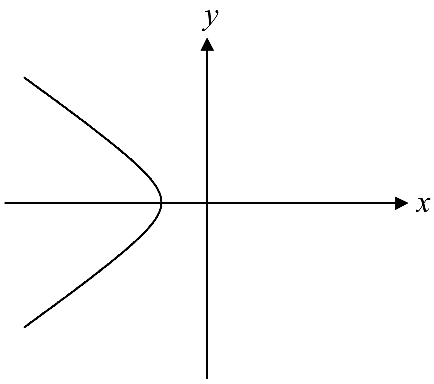

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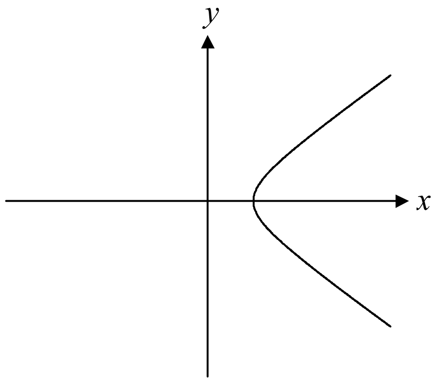

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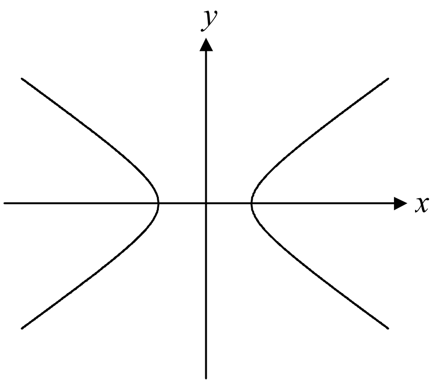


Question 6

Consider the function $f: R \to R$ where $f(x) = 4x^3 - 3x^4$

Which one of the following statements is not true?

A. f has two stationary points

B. f has two points of inflexion

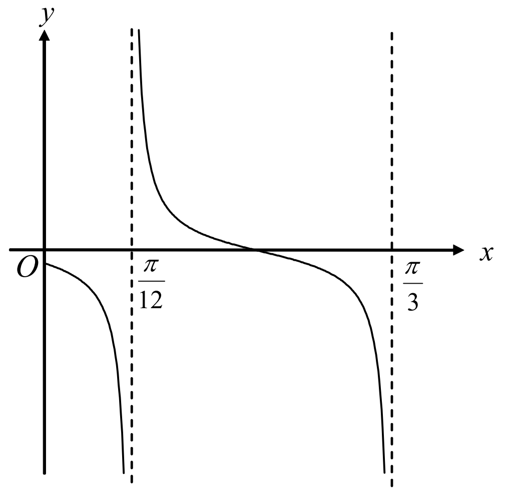
C. f' is maximum when $x = \frac{2}{3}$

D. $\frac{1}{f}$ has three asymptotes

E. $f = \frac{1}{f}$ has three solutions

Question 7

A graph of $f: \left[0, \frac{\pi}{3}\right)$ where $f(x) = \cot\left(nx - \frac{\pi}{3}\right)$ is sketched below.



The value of n could be

A. $\frac{1}{4}$

B. $\frac{1}{3}$

C. 3

D. 4

E. 8

Question 8

The gradient of the curve $y^2 = 4x + 6y - 5$ is $-\frac{2}{3}$ at the point where y equals

A. 0

B. 0.15

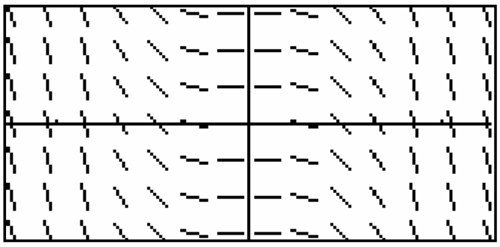
C. 1.25

D. 5

E. 6

Question 9

The slope field from a first order differential equation is shown below.



If $a \in R$, a solution of this differential equation could be

A. $y = a\log_e(x)$

B. $y = a\cos(x)$

C. $y = a\tan^{-1}(x)$

D. $y = \frac{a}{x^2}$

E. $y = ax^3$

Question 10

Given $\dfrac{dy}{dx} = \sqrt{\sin(2x)}$ and $y = \sqrt{2}$ when $x = \dfrac{\pi}{12}$.

The value of y when $x = \dfrac{\pi}{3}$ is

A. 0.2500

B. 0.7298

C. 0.9306

D. 1.4369

E. 2.1440

Question 11

Using a suitable substitution, $\int_5^{10} \dfrac{1}{x^2} e^{\frac{10}{x}}\, dx$ can be expressed as

A. $\int_1^2 \dfrac{100}{u^2} e^u\, du$

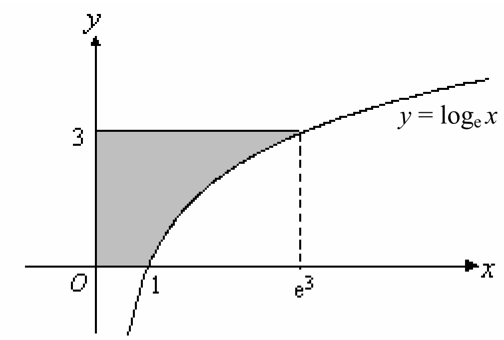
B. $100 \int_1^2 u^2 e^u\, du$

C. $-10 \int_2^1 e^u\, du$

D. $\dfrac{1}{10} \int_1^2 e^u\, du$

E. $-\dfrac{1}{10} \int_5^{10} e^u\, du$

Question 12



The graph of $y = \log_e x$ is shown above. The volume of the solid of revolution formed when the shaded region is rotated around the y-axis is given by

A. $\pi\int_0^3 (3 - \log_e x)^2\, dx$

B. $\pi\int_1^{e^3} (\log_e x)^2\, dx$

C. $\pi\int_1^{e^3} (3 - e^y)^2\, dy$

D. $\pi\int_0^3 e^y\, dy$

E. $\pi\int_0^3 e^{2y}\, dy$

Question 13

A spherical ice ball initially has radius 0.9 cm. It is placed in a drink and melts at a constant rate of 1.5 cm^3/minute. When the radius is 0.6 cm, the rate, in cm/minute, at which the radius is decreasing is

A. $\frac{5}{24\pi}$

B. $\frac{25}{72\pi}$

C. $\frac{25}{24\pi}$

D. $\frac{54\pi}{25}$

E. $\frac{36\pi}{25}$

Question 14

A tank initially contains 200 litres of pure water. A salt solution with a concentration of 0.2 kg/litre is poured into the tank at a rate of 5 litres/minute. The mixture is kept uniform by stirring and flows out of the tank at a rate of 3 litres/minute.

Let Q be the amount of salt in the tank after t minutes.

$\frac{dQ}{dt}$ is equal to

A. $5-\frac{3Q}{200+2t}$

B. $5-\frac{3Q}{200}$

C. $(5-3t)\frac{Q}{200}$

D. $1-\frac{3Q}{200-2t}$

E. $1-\frac{3Q}{200+2t}$

Question 15

Let $\underset{\sim}{u} = 6\underset{\sim}{i} + 2\underset{\sim}{j} - 3\underset{\sim}{k}$ and $\underset{\sim}{v} = 2\underset{\sim}{i} - \underset{\sim}{j} + 3\underset{\sim}{k}$.

The vector resolute of $\underset{\sim}{u}$ in the direction of $\underset{\sim}{v}$ is

A. $\frac{1}{49}(2\underset{\sim}{i} - \underset{\sim}{j} + 3\underset{\sim}{k})$

B. $\frac{1}{7}(2\underset{\sim}{i} - \underset{\sim}{j} + 3\underset{\sim}{k})$

C. $\frac{1}{14}(2\underset{\sim}{i} - \underset{\sim}{j} + 3\underset{\sim}{k})$

D. $\frac{1}{\sqrt{14}}(2\underset{\sim}{i} - \underset{\sim}{j} + 3\underset{\sim}{k})$

E. $\frac{1}{7\sqrt{14}}(2\underset{\sim}{i} - \underset{\sim}{j} + 3\underset{\sim}{k})$

Question 16

Points A, B and C are collinear such that $AB : BC = 1 : 3$

If $\overrightarrow{OA} = \underset{\sim}{a}$ and $\overrightarrow{OC} = \underset{\sim}{c}$ then $\overrightarrow{OB}$ equals

A. $\frac{1}{4}(3\underset{\sim}{a} + \underset{\sim}{c})$

B. $\frac{1}{4}(\underset{\sim}{a} + 3\underset{\sim}{c})$

C. $\frac{1}{4}(5\underset{\sim}{a} - \underset{\sim}{c})$

D. $\frac{1}{3}(2\underset{\sim}{a} + \underset{\sim}{c})$

E. $\frac{1}{3}(\underset{\sim}{a} - 3\underset{\sim}{c})$

Question 17

The position of a particle at time t is given by $\underset{\sim}{r}(t) = (t^3 + 2t)\underset{\sim}{i} + 5t\underset{\sim}{j} - t^2\underset{\sim}{k}$.

The magnitude of its acceleration when $t = 1$ is

A. $3\underset{\sim}{i} + 5\underset{\sim}{j} - \underset{\sim}{k}$

B. $6\underset{\sim}{i} - 2\underset{\sim}{k}$

C. $2\sqrt{10}$

D. $3\sqrt{6}$

E. $\sqrt{35}$

Question 18

A particle is moving in a straight line with an acceleration of $-20x + 20$ m/s^2, where x is its displacement, in metres, from a fixed point O. If the particle is travelling with a velocity of 6 m/s when it is 3 metres to the right of O, its maximum speed, in m/s, is

A. 6.0

B. 9.8

C. 10.0

D. 10.8

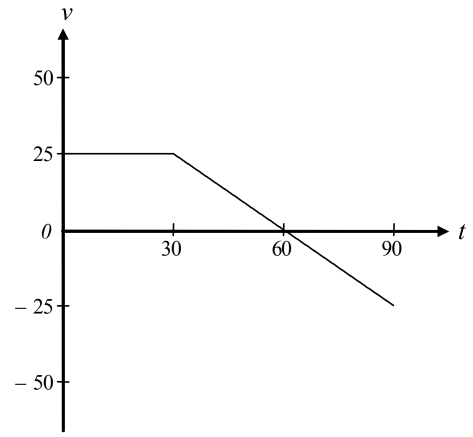
E. 12.0

Question 19

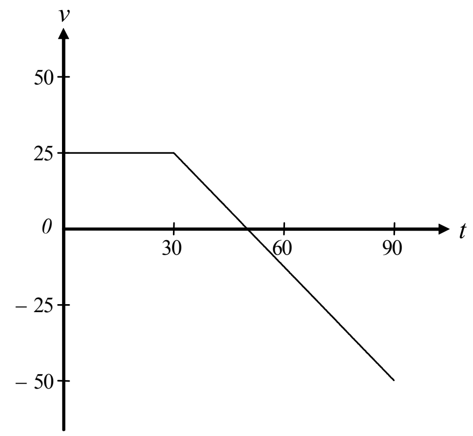
A particle travels in a straight line with a constant velocity of 25 m/s for 30 seconds.
It then decelerates for 60 seconds and returns to its original position.

The velocity-time graph that best represents the motion of the particle is

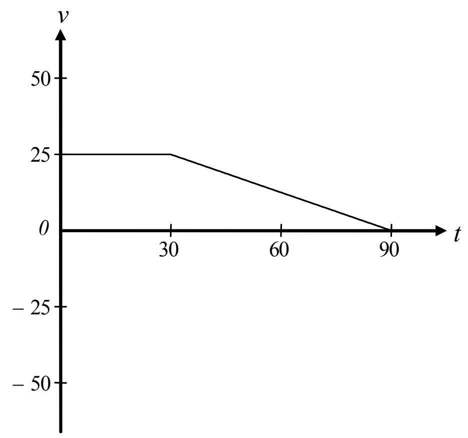
A.



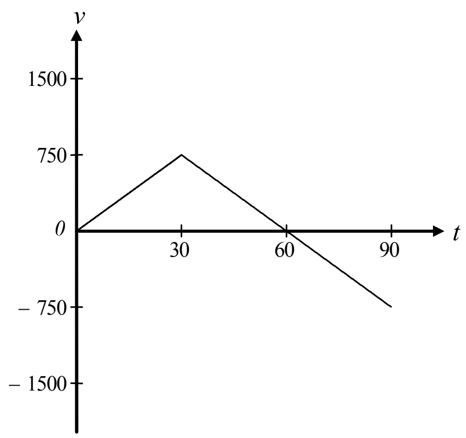
B.



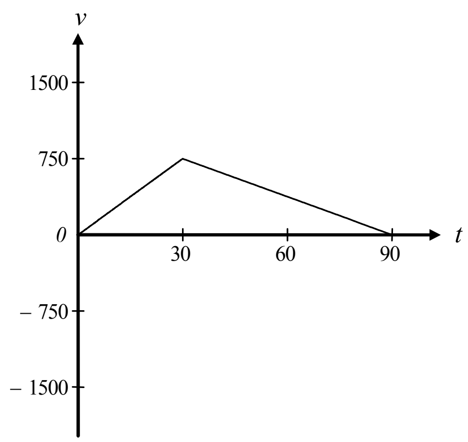
C.



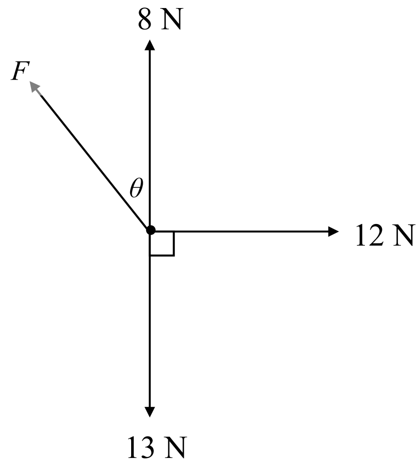
D.



E.



Question 20



Four forces are acting on a particle as shown in the diagram above.

The particle will be in equilibrium when F, measured in newtons, is equal to

A. $5\cos\theta$

B. $12\sin\theta$

C. $\dfrac{\cos\theta}{12}$

D. 5

E. 13

Question 21

A motorbike is travelling at a speed of 60 km/hr on a straight road. A school zone is observed in the distance and over the next 10 seconds it reduces speed to 40 km/hr.

If the mass of the motorbike is 900 kg, the change in momentum, measured in kg m/s, in the direction of motion is

A. −6480

B. −5000

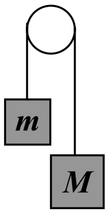
C. −1800

D. −500

E. −180

Question 22

A mass of m kg is attached to a second mass of M kg, $m < M$, by a light string passing over a smooth pulley as shown below. The tension in the string is T newtons.



The acceleration, in m/s^2, of the M kg mass is

A. g

B. Mg

C. $\dfrac{Mg - T}{m}$

D. $\dfrac{g(M - m)}{(M + m)}$

E. $\dfrac{g(M + m)}{(M - m)}$

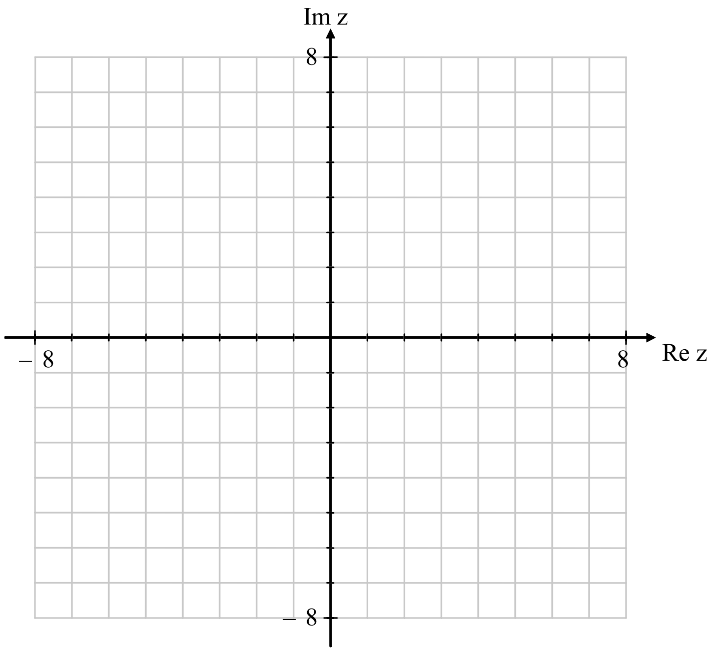
END OF SECTION 1

SECTION 2

Instructions for Section 2

Answer all the questions in the spaces provided.
A decimal approximation will not be accepted if an exact answer is required to a question.
In questions where more than one mark is available, approximate working must be shown.
Unless otherwise indicated, the diagrams in this book have not been drawn to scale.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$

Question 1



a. Let $P = 4\sqrt{2}\text{cis}\left(\frac{\pi}{4}\right)$.

Express P in Cartesian form and plot and label this point in the Argand plane above.

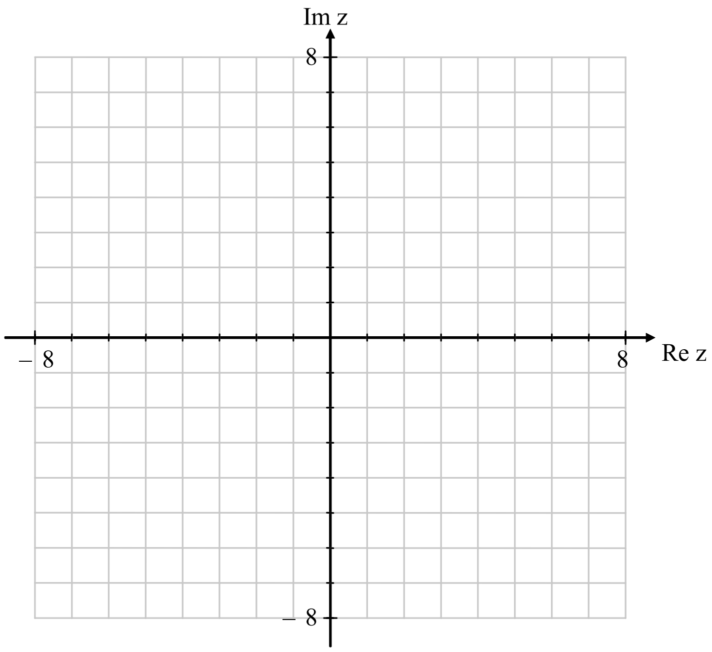
1 mark

b. **i.** Find an equivalent Cartesian equation for

$$\{z : |z + 2 - 4i| = |z - 2|, z \in C\}$$

2 marks

ii. Hence sketch $\{z : |z + 2 - 4i| = |z - 2|, z \in C\}$ on the Argand plane below.



1 mark

c. Describe the key features of the relation defined by $\{z : |z - i| = 5\}$

2 marks

d. *M* and *N* are the points of intersection of the relations $\{z : |z - i| = 5\}$ and $\{z : |z + 2 - 4i| = |z - 2|\}$. Determine points *M* and *N* in Cartesian form using your graphics calculator.

2 marks

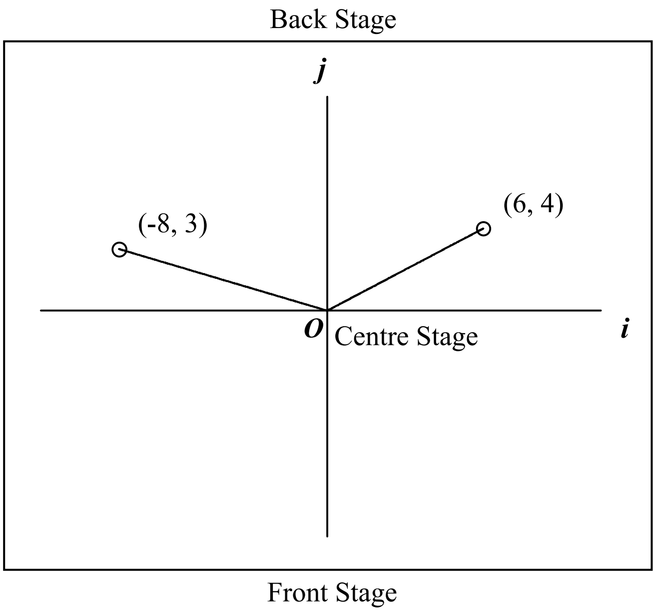
e. Use vectors to prove that points *M*, *N* and *P* are the vertices of a right-angled triangle.

3 marks

Total 11 marks

Question 2

Two dancers, Ari, A, and Ben, B, are standing on stage at the start of a performance. Their position coordinates, in metres, in relation to point O at the centre of the stage are shown in the diagram below.



a. Write vectors $\overrightarrow{OA}$ and $\overrightarrow{OB}$ in terms of $\underset{\sim}{i}$ and $\underset{\sim}{j}$ to describe the positions of Ari and Ben at the start of the performance.

1 mark

b. Find the obtuse angle AOB in degrees correct to one decimal place.

2 marks

As the performance starts spotlight, *r*, is beamed onto the stage. The path the spotlight follows around the stage is given by the equation $\underset{\sim}{r} = 10\cos(t)\,\underset{\sim}{i} + 5\sin(t)\,\underset{\sim}{j}, \quad t \geq 0$.

c. Write a vector that describes the position of spotlight *r* initially.

1 mark

d. Show that both Ari and Ben are standing in the path traced out by spotlight *r*.

3 marks

e. How long after the spotlight passes Ari does it reach Ben? Write your answer in seconds correct to two decimal places.

2 marks

A second spotlight, s, starts moving at the same time as spotlight r. It follows a path given by the equation $\underset{\sim}{s} = 5\sin(t)\,\underset{\sim}{i} + 10\cos(t)\,\underset{\sim}{j}, \quad t \geq 0$.

f. Find the times and position coordinates of the points on stage where the spotlights meet. Write your answers correct to two decimal places.

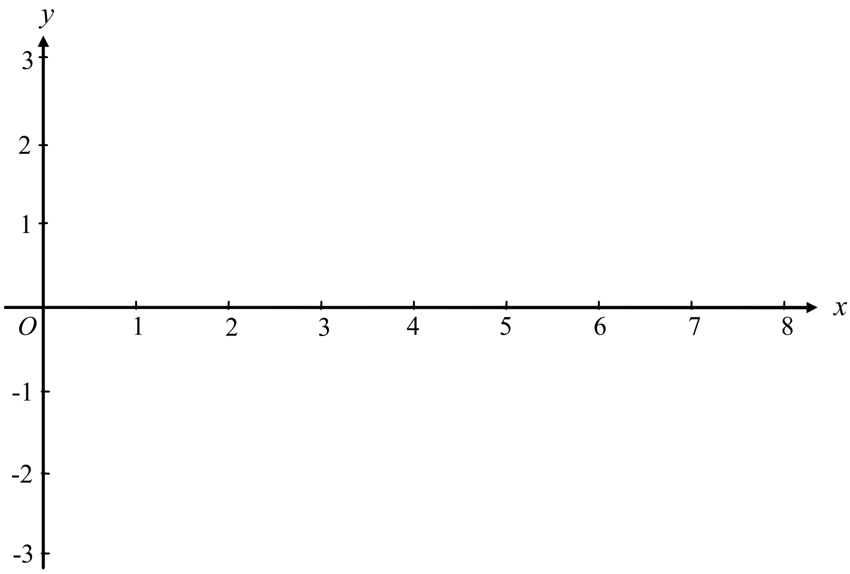
3 marks

Total 12 marks

Question 3

Consider the function $f : D \to R$ where $f(x) = 0.5\text{cosec}\left(\frac{\pi}{4}(2 - x)\right)$

a. **i.** On the axes below, sketch a graph of *f* over the interval $[0, 8]$, labelling all features clearly.



2 marks

ii. Determine the domain and range of f over this interval.

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2 marks

b. An equivalent rule for f is $f_1(x) = \dfrac{1}{a\cos(bx + c)}$ where $a, b, c \in R$

Give values for a, b, and c.

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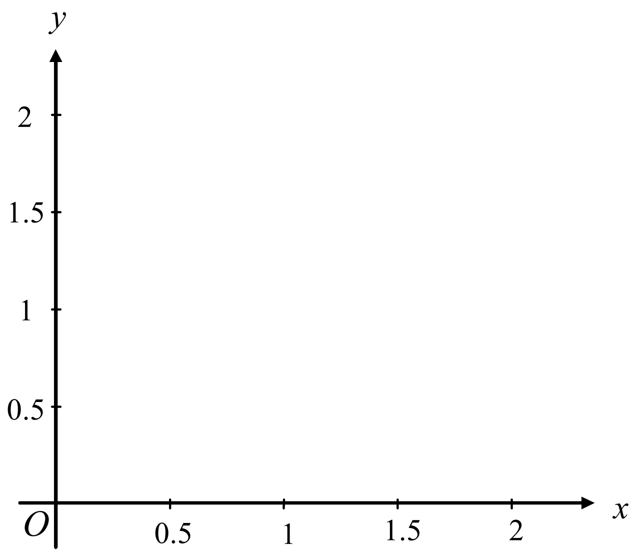
2 marks

c. Let $D = [0, 2)$.

Sketch f and f^{-1} on the axes below, clearly showing the key features.



1 mark

d. Write a definite integral that will give the area enclosed by f and f^{-1}. Using your graphics calculator, evaluate this integral correct to three decimal places.

3 marks

Total 10 marks

Question 4

A box of mass m kg is dropped from a hot air balloon. Its motion is retarded by a variable force of $\frac{mv}{5}$ newton, where v m/s is the velocity of the box t seconds after it is dropped.

a. Taking vertically downwards as positive, show that the differential equation $\frac{dv}{dt} = \frac{5g - v}{5}$, where $g = 9.8$ m/sec^2 is the acceleration due to gravity, applies to this situation.

2 marks

b. Hence, show that $t = 5\log_e\left(\frac{5g}{5g - v}\right)$

2 marks

c. Show that at time t the velocity of the box is $5g\left(1 - e^{-0.2t}\right)$ m/s.

2 marks

d. Write an expression for the limiting velocity of the box. Show how you deduced your result.

2 marks

e. Determine the time taken for the box to reach half its limiting velocity. Write your answer in seconds correct to two decimal places.

2 marks

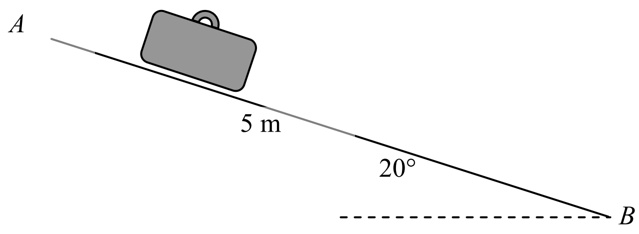
f. Find the distance travelled by the box in the first 10 seconds of motion. Write your answer correct to the nearest metre.

3 marks

Total 13 marks

Question 5

Baggage handlers use ramps to transport luggage. Ramp *AB* is 5 metres in length and inclined at an angle of 20° to the horizontal. A 20 kg suitcase, initially at rest at *A*, slides down ramp *AB* under the force of gravity. The coefficient of friction between the suitcase and the ramp is 0.2. Take $g = 9.8$ m/sec^2.



a. On the diagram above, draw all forces acting on the suitcase as it slides down the ramp.

1 mark

b. Show that the suitcase slides down the ramp with an acceleration of 1.51 m/s^2.

2 marks

c. Find the time taken for the suitcase to reach point *B*. Write your answer in seconds correct to two decimal places.

2 marks

d. Some time later, an identical 20 kg suitcase, initially at rest at *A*, is pushed down the ramp with a force of $100 - 200t$ newtons for the first 0.5 seconds of motion. Show that at time *t*, $0 < t < 0.5$, the acceleration of this suitcase is $6.51 - 10t$ m/s^2.

2 marks

e. Find the speed of the suitcase when $t = 0.5$. Write your answer in m/s, correct to two decimal places.

2 marks

f. Determine the speed of this suitcase when it reaches point *B*. Write your answer in m/s, correct to two decimal places.

3 marks

12 marks

END OF QUESTION AND ANSWER BOOK